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VORTEX DIPOLE RESPONSE IN THE GIANT DIPOLE RESONANCE ENERGY REGION

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The velocity fields associated with isovector excitations of spherical nuclei in the giant dipole resonance (GDR) energy region have been studied within a semiclassical approach based on the solution of the Vlasov kinetic equation for finite two-component Fermi systems with a moving surface. The neutron-proton asymmetry and dynamical surface effects lead to the fragmentation of the isovector dipole strength in the energy region of the GDR on two resonances. It was found that the velocity field has a potential character in the energy range near the main (low-energy) maximum of the GDR. However, the velocity field reveals a vortex character in the surface region at the energy of the high-energy maximum of the GDR.

1. Introduction

In order to obtain more the information concerning the nature of collective excitations in nuclei, it is expedient to consider velocity fields. The latter describe the spatial distribution of the average velocity of nucleons at their collective motion. They can be determined in both quantum-mechanical [1 - 5] and semiclassical (macroscopic) approaches [6 - 8], which makes a direct comparison between them possible.

In this paper the velocity fields for isovector dipole excitations are considered in the framework of the kinetic model [8]. Such semiclassical approach provides an exact solution of the problem of separating the spurious centre of mass motion from the intrinsic excitations for isovector dipole response of neutron-proton asymmetric systems. Within this semiclassical approach, it is found that the isovector dipole response function has two peaks in the energy range of giant isovector resonance in nuclei [9]. It is of interest to study the velocity fields for resonances found in work [9], in order to elucidate the origin of those collective excitations and to compare them with the results of quantum-mechanical calculations. In Section 2 we remind a formalism of the kinetic model and will find the response function. In Section 3 the velocity fields for isovector dipole excitations are considered within our approach and the results of numerical calculations of the velocity fields associated with isovector dipole excitations are presented.

2. The kinetic model

We use a semiclassical theory of linear response of finite Fermi systems based on the Vlasov kinetic equation for two-component systems [9] for the study of the velocity fields associated with collective dipole modes in spherical nuclei. At first we review the general formalism of our kinetic approach. We consider a finite Fermi system consisting of neutrons and protons and confined by a moving spherical surface

$$R_q(\theta,\varphi,t) = R + \delta R_q(\theta,\varphi,t). \tag{1}$$

Here $q = n, p$ and R is the equilibrium radius. The amplitudes $\delta R_q(\theta,\varphi,t)$ give the dynamical change of the equilibrium radius.

We assume that the isovector dipole excitations of our system can be described by the phase-space distribution function $\delta n_q(\vec{r},\vec{p},t)$ given by the linearized Vlasov equation:

$$\frac{\partial}{\partial t}\delta n_q(\vec{r},\vec{p},t) + \frac{\vec{p}}{m}\frac{\partial}{\partial \vec{r}}\left[\delta n_q(\vec{r},\vec{p},t) - \frac{dn_q^0(\varepsilon)}{d\varepsilon_q}\left[\delta U_q(\vec{r},t) + V_q(\vec{r},t)\right]\right] = 0$$

$$(r < R) \tag{2}$$

with mirror reflection boundary conditions at the moving surface

$$[\delta n_q(\vec{r},\vec{p}_\perp,p_r,t) - \delta n_q(\vec{r},\vec{p}_\perp,-p_r,t)]\big|_{r=R} =$$

$$= -2p_r\frac{dn_q^0(\varepsilon)}{d\varepsilon}\frac{\partial}{\partial t}\delta R_q(\theta,\varphi,t), \tag{3}$$

where $\vec{r}$ and $\vec{p}$ are the radius-vector and the momentum of the particle, respectively; p_r is the radial momentum and $\vec{p}_\perp = (0, p_\theta, p_\phi)$; $n_q^0(\varepsilon)$ are equilibrium distribution functions and $\frac{dn_q^0(\varepsilon)}{d\varepsilon} =.$ $= -\delta(\varepsilon - \varepsilon_F^q)$. Here the Fermi energy of the neutron system at $q = n$ (or the proton system at $q = p$) is given by $\varepsilon_F^q = \varepsilon_F \left(1+\tau_q \frac{N-Z}{A}\right)^{2/3}$, where ε_F is the Fermi energy of nuclear matter.

We study the response of our system on the isovector dipole external field defined as

$$V_q(\vec{r},t) = \beta\delta(t) a_q r Y_{10}(\theta). \tag{4}$$

Here $a_q = \frac{2Z}{A}$ at $q=n$ and $a_q = -\frac{2N}{A}$ at $q=p$, so that the external field $V_q(\vec{r},t)$ generates the motion of the protons and neutrons against each other.

In order to take into account the main effects of the mean field change inside the system, it was assumed that they can be reduce to the separable residual interaction between the nucleons given by

$$u_{qq'}(\vec{r},\vec{r}') = \kappa_{qq'} \sum_M rr' Y_{1M}(\theta,\varphi) Y_{1M}^*(\theta',\varphi') \tag{5}$$

with the strength parameters $\kappa_{qq'}$

$$\kappa_{nn} = \kappa_{pp} = \frac{40\pi}{9A}\frac{\varepsilon_F}{R^2}(F_0 + F_0'),$$

$$\kappa_{np} = \kappa_{pn} = \frac{40\pi}{9A}\frac{\varepsilon_F}{R^2}(F_0 - F_0'). \tag{6}$$

Here F_0' and F_0 are isovector and isoscalar parameters Landau, respectively.

To solve the kinetic equation (2) we take the Fourier transform with respect to time and perform a transition from the variables $(\vec{r},\vec{p})$ to new variables $(r,\varepsilon,l,\alpha,\beta,\gamma)$ [8]. Here, ε is the particle's energy; l its angular moment; the Euler angles α,β,γ determine the rotation to a coordinate system, whose z-axis is directed along the vector $\vec{l} = |\vec{r}\times\vec{p}|$ and y-axis along the vector $\vec{r}$. Then the solution of the Vlasov equation (2), satisfying the moving-boundary condition (3), can be presented in the following form:

$$\delta n_q(r,\varepsilon,l,\omega) = \sum_N \left[\delta\tilde{n}^+_{q,N}(r,\varepsilon,l,\omega)\theta(p_r) + \delta\tilde{n}^-_{q,N}(r,\varepsilon,l,\omega)\theta(-p_r)\right]\left(D^1_{0N}(\alpha,\beta,\gamma)\right)^* \tag{7}$$

with

$$\delta\tilde{n}^\pm_{q,N}(r,\varepsilon,l,\omega) = \delta n^{0\pm}_{q,N}(r,\varepsilon,l,\omega)\left(1+\sum_{q'=n,p}\kappa_{qq'}\frac{\tilde{R}^v_{q'}(\omega)}{a_q a_{q'}}\right) + \delta\tilde{n}^{s\pm}_{q,N}(r,\varepsilon,l,\omega). \tag{8}$$

Here the function $\delta n^{0\pm}_{q,N}(r,\varepsilon,l,\omega)$ is the solution of the Vlasov equation for a system of noninteracting nucleons confined by the fixed surface [10]

$$\delta n^{0\pm}_{q,N}(r,\varepsilon,l,\omega) = -\beta\frac{\partial n^0_q(\varepsilon)}{\partial\varepsilon}\sum_{n=-\infty}^{\infty}\omega_{nN}(\varepsilon,l)e^{\pm i[\omega_{nN}(\varepsilon,l)\tau(r,\varepsilon,l) - N\gamma(r,\varepsilon,l)]} \times \frac{Q_q(nN,\varepsilon l)}{\omega - \omega_{nN}(\varepsilon,l) + i\eta}, \tag{9}$$

where $Q_q(nN,\varepsilon\, l) = (-1)^n \frac{a_q v_F^q}{R}\frac{1}{\omega^2_{nN}(\varepsilon,l)}$ is the classical limit for the radial matrix elements of the quantum-mechanical dipole operator, $\omega_{nN}(\varepsilon,l) = n\frac{2\pi}{T(\varepsilon,l)} + N\frac{\Gamma(\varepsilon,l)}{T(\varepsilon,l)}$ are single-particle frequencies, and $T(\varepsilon,l)$, $\Gamma(\varepsilon,l)$ are the periods of the radial and "angular" particle motions.

The second summand on the right-hand side of Eq. (8) $\delta\tilde{n}^{s\pm}_{q,N}(r,\varepsilon,l,\omega)$ is an extra term in the solution of the Vlasov equation, which is associated with the moving surface,

$$\delta\tilde{n}^{s\pm}_{q,N}(r,\varepsilon,l,\omega) = \frac{\partial n^0_q(\varepsilon)}{\partial\varepsilon}\frac{e^{\pm i[\omega\tau(r,\varepsilon,l) - N\gamma(r,\varepsilon,l)]}}{\sin[\omega T(\varepsilon,l) + N\Gamma(\varepsilon,l)]}p(R,\varepsilon,l)\omega\delta R_q(\omega). \tag{10}$$

Here $\delta R_q(\omega)$ *are* the changes of the equilibrium radius R of the system induced by the external field (4), $2\tau(r,\varepsilon,l)\big|_{r=R} = T(\varepsilon,l)$ and $2\gamma(r,\varepsilon,l)\big|_{r=R} = \Gamma(\varepsilon,l)$.

The function $\tilde{R}^v_q(\omega)$ in Eq. (8) determines the variation of the mean field caused by a residual separable interaction in the system bulk [9].

We are interested in the response function which is determined in the following way:

$$\tilde{R}\ (\omega) = \sum_{q=n,p} \tilde{R}_q(\omega) = \frac{1}{\beta}\sum_{q=n,p}\int d\vec{r} a_q r Y_{10}(\theta)\delta\tilde{\rho}_q(\vec{r},\omega)\,, \tag{11}$$

where $\delta\tilde{\rho}_q(\vec{r},\omega)$ is the Fourier transform with respect to time of the density change of neutrons (or protons) induced by the external field $V_q(\vec{r},t)$. In the moving-surface approximation the density change $\delta\tilde{\rho}_q(\vec{r},\omega)$ is defined as

$$\delta\tilde{\rho}_q(\vec{r},\omega) = \frac{2}{h^3}\int d\vec{p}\delta\tilde{n}_q(\vec{r},\vec{p},\omega) + $$
$$+\delta(r-R)\rho_q\delta R_q(\omega)Y_{10}(\theta)\,, \tag{12}$$

where ρ_q is the neutron (or proton) equilibrium density.

Substituting the distribution function (8) into Eq. (12), the collective response function (11) can be written as

$$\tilde{R}(\omega) = R(\omega) + \tilde{S}(\omega)\,. \tag{13}$$

Here $R(\omega) = \sum_{q=n,p} R_q(\omega)$ is the collective dipole response function in the fixed-surface approximation [9]. The function $\tilde{S}(\omega,T)$ in Eq. (13) represents the moving-surface contribution [9].

The isovector dipole strength function associated with the found response function is given by

$$S(E) = -\frac{1}{\pi}\operatorname{Im}\tilde{R}(E)\,, \tag{14}$$

where $E = \hbar\omega$.

The strength function (14) describes the strength distribution of isovector dipole excitations.

3. Velocity field for isovector dipole modes

In order to obtain more the information concerning the nature of collective excitations in nuclei we will consider velocity fields. At equilibrium the average velocity of particles in the systems (the velocity field) is equal to zero. If the system is embedded into a weak external field, the velocity field will be different from zero. In the kinetic theory the Fourier-transform with respect to time of the velocity field in the linear approximation is determined as

$$\vec{u}(\vec{r},\omega) = \sum_{q=n,p}\tau_q\vec{u}_q(\vec{r},\omega)\,, \tag{15}$$

$$\vec{u}_q(\vec{r},\omega) = \frac{1}{m\rho_0}\int d\vec{p}\,\vec{p}\,\delta\tilde{n}_q(\vec{r},\vec{p},\omega)\,, \tag{16}$$

where $\tau_n = 1,\ \tau_p = -1$ and ρ_0 is the nuclear matter density at equilibrium.

The velocity field will be considered in the *XZ* coordinate plane ($\vec{r} = (x, y=0, z)$ or in the spherical coordinates $\vec{r} = (r,\theta,\varphi=0)$) because such representation is used in quantum-mechanical approaches of the RPA type [3]. In this case the velocity field can be written as

$$\vec{u}_q(r,\theta,\varphi=0,\omega) = u_z^q(r,\theta,\omega)\vec{e}_z + u_x^q(r,\theta,\omega)\vec{e}_x\,, \tag{17}$$

where $u_x^q(r,\theta,\omega)$ and $u_z^q(r,\theta,\omega)$ are the projections of the velocity field vector onto the X and Z axes, respectively; $\vec{e}_x, \vec{e}_z$ are unit vectors directed along these axes.

Using the kinetic approach for the description of collective excitations in finite Firm systems [8], the projections $u_x^q(r,\theta,\omega)$ and $u_z^q(r,\theta,\omega)$ of the velocity field for dipole excitations can be written down, after some transformations, in the form

$$u_z^q(r,\theta,\omega) = Y_{00}(\theta,0)u_{10}^q(r,\omega) - \sqrt{\frac{2}{5}}Y_{20}(\theta,0)u_{12}^q(r,\omega)\,, \tag{18}$$

$$u_x^q(r,\theta,\omega) = \sqrt{\frac{3}{5}}Y_{21}(\theta,0)u_{12}^q(r,\omega)\,. \tag{19}$$

The angular dependence of the velocity field is expressed in terms of spherical functions and coincides with the dependence obtained in quantum-mechanical approaches of the RPA type (see work [3]). The functions $u_{10}^q(r,\omega)$ and $u_{12}^q(r,\omega)$ in Eqs. (18) and (19) describe the radial dependence of the velocity field. The following expressions can be obtained for them:

$$u_{12}^q(r,\omega) = -i\sqrt{\frac{2}{3}}\pi\frac{1}{\rho_0}\frac{1}{r^2}\int d\varepsilon\int dl\, l\sum_{N=-1}^{1}\Big\{-i[\delta\tilde{n}_{q,N}^{+}(r,\varepsilon,l,\omega) - \delta\tilde{n}_{q,N}^{-}(r,\varepsilon,l,\omega)] + $$
$$+\frac{N}{2}\frac{l}{p_r(r,\varepsilon,l)\,r}[\delta\tilde{n}_{q,N}^{+}(r,\varepsilon,l,\omega) + \delta\tilde{n}_{q,N}^{-}(r,\varepsilon,l,\omega)]\Big\} \tag{20}$$

and

$$u_{10}^{q}(r,\omega) = -i\sqrt{\frac{1}{3}}\pi\frac{1}{\rho_0}\frac{1}{r^2}\int d\varepsilon\int dl\, l\sum_{N=-1}^{1}\left\{i[\delta\tilde{n}_{q,N}^{+}(r,\varepsilon,l,\omega) - \delta\tilde{n}_{q,N}^{-}(r,\varepsilon,l,\omega)] + \right.$$
$$\left. + N\frac{l}{p_r(r,\varepsilon,l)\,r}[\delta\tilde{n}_{q,N}^{+}(r,\varepsilon,l,\omega) + \delta\tilde{n}_{q,N}^{-}(r,\varepsilon,l,\omega)]\right\}. \quad (21)$$

Thus, using the expressions given above, one can carry out numerical calculations of the velocity field in the *XZ* plane in the case of isovector dipole excitations, see Eqs. (18) and (19).

The numerical calculations of the strength function *S(E)* (14) and the dipole velocity field, see Eqs. (18) and (19), were carried out for an asymmetric system with the neutron number N = = 126 and the proton number Z = 82. In the course of calculations, the following standard values of nuclear parameters were used: r_0= 1.12 fm, ε_F = = 40 MeV, m = 1.04 MeV $(10^{-22}$ s$)^2$/fm^2. The values for the surface symmetry energy parameter Q and the Landau parameters F_0 and F_0' were taken from work [11]: Q = 75 MeV, F_0' = 1.25, and F_0 = -0.42.

In Fig. 1 the strength function for isovector dipole excitations *S(E)* given by Eqs. (13) and (14) is shown. One can see that the isovector dipole strength distribution has two maxima at energies of 12.2 and 14.6 MeV. The low-energy maximum describes the isovector giant dipole resonance in heavy nuclei. The high-energy maximum is generated in neutron-rich nuclei when the dynamic surface effects are taken into account.

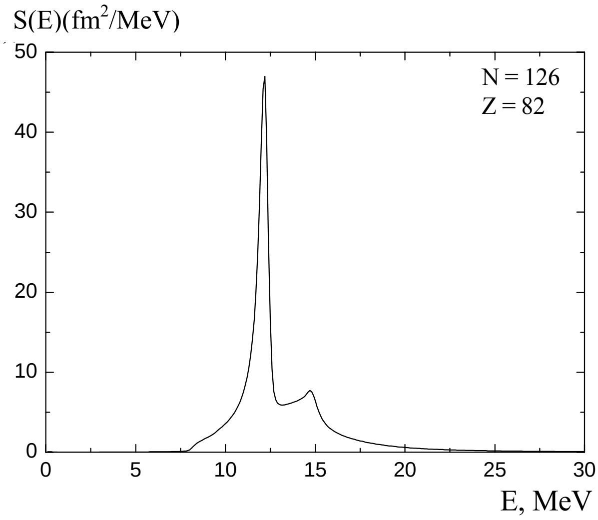
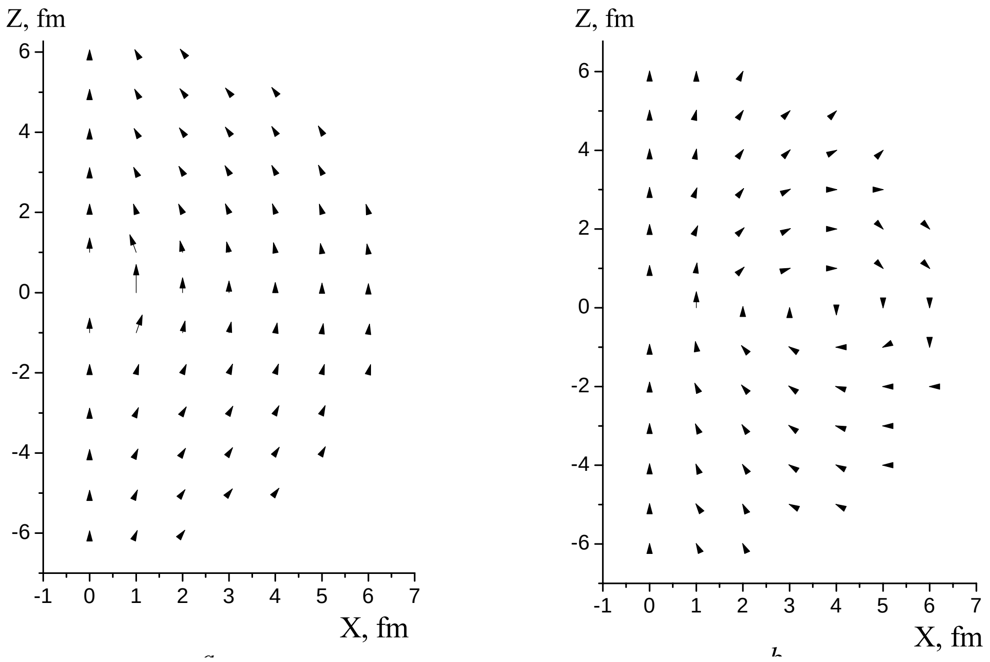


Fig. 1. Strength distribution of isovector dipole excitations in an asymmetric system with the neutron number N = 126 and the proton number Z = 82.

In Fig. 2 the results of numerical calculations of the isovector dipole velocity fields are represented. Fig. 2 exhibits the velocity fields calculated for the energies corresponding to low-and high-energy maxima (see Fig. 1). It is evident, see Fig. 2, *a*, that



a *b*

Fig. 2. Velocity fields in the XZ plane of an asymmetric system with the neutron number N = 126 and the proton number Z = 82 calculated for the isovector giant dipole resonance, in the moving-surface approximation, and at energies E_{max} = 12.2 (*a*) and 14.6 MeV (*b*) which correspond to the energies of the low- and high-energy resonance strength maxima, respectively (see Fig. 1).

the velocity field at the energy of the low-energy maximum is the motion of particles along the Z-axis (along the direction of action of the external force (3)) in the nucleus bulk. Small deviations from the motion along the z-axis are observed only in the near-surface region, due to the mirror reflection boundary conditions at the moving surface. Such behavior of the velocity field is in agreement with the results obtained in the Goldhaber - Teller model [1, 5].

However, in contrast to the Goldhaber - Teller model, the radial component of the velocity field $u_{12}^{q}(r,\omega)$ can differ from zero in our model, see Eq. (8). Really, at the energy of the high-energy maximum, the component $u_{12}^{q}(r,\omega)$ is equal to the component $u_{10}^{q}(r,\omega)$ in the near-surface region. The velocity field at this energy has a vortex character in this region (see Fig. 2, *b*). The isovector dipole velocity fields obtained in this work agree with the results of corresponding quantum-mechanical calculations carried out in the RPA framework [1, 3].

4. Conclusions

The strength function and the velocity fields for isovector dipole excitations in spherical nuclei have been studied in a semiclassical approach, which is based on the Vlasov kinetic equation for a finite two-component system with moving surface. The expression for the velocity field in the *XZ* coordinate plane, presented in terms of the particle distribution function in the phase space, has been considered. The analytical expression for the velocity field has been derived in the case of collective isovector dipole excitations. Numerical calculations have been carried out for the strength function and the velocity fields associated with isovector excitations in the GDR energy region. The neutron-proton asymmetry leads to the fragmentation of the isovector dipole strength in the energy region of the GDR on two resonances. The low-energy resonance reproduces the GDR in heavy nuclei. The resonance at higher energy is generated in neutron-proton asymmetric nuclei by the surface symmetry potential. It is found that the velocity field associated with low-energy resonance has a potential character. However, the velocity field corresponding to high-energy resonance reveals a vortex character.

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ВИХРОВИЙ ДИПОЛЬНИЙ ВІДГУК В ОБЛАСТІ ЕНЕРГІЙ ГІГАНТСЬКОГО ДИПОЛЬНОГО РЕЗОНАНСУ

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Поля швидкостей, що пов’язані з ізовекторними збудженнями сферичних ядер в області енергій гігантського дипольного резонансу (ГДР), вивчались у рамках напівкласичного підходу, що спирається на розв’язок кінетичного рівняння Власова для скінченних двокомпонентних фермі-систем з рухомою поверхнею. Нейтрон-протонна асиметрія та динамічні поверхневі ефекти призводять до фрагментації ізовекторної дипольної сили в області енергій ГДР на два резонанси. Встановлено, що поле швидкостей має потенціальний характер в області енергій головного (низькоенергетичного) максимуму ГДР. Проте поле швидкостей виявляє вихровий характер у поверхневій області при енергії високоенергетичного максимуму ГДР.

ВИХРЕВОЙ ДИПОЛЬНЫЙ ОТКЛИК В ОБЛАСТИ ЭНЕРГИЙ ГИГАНТСКОГО ДИПОЛЬНОГО РЕЗОНАНСА

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Поля скоростей, связанные с изовекторными возбуждениями сферических ядер в области энергий гигантского дипольного резонанса (ГДР), изучались в рамках полуклассического подхода, который опирается на решение кинетического уравнения Власова для конечных двухкомпонентных ферми-систем с подвижной поверхностью. Нейтрон-протонная асимметрия и динамические поверхностные эффекты приводят к фрагментации изовекторной дипольной силы в области энергий ГДР на два резонанса. Установлено, что поле скоростей имеет потенциальный характер в области энергий главного (низкоэнергетического) максимума ГДР. Однако поле скоростей обнаруживает вихревой характер в поверхностной области при энергии высоко-энергетического максимума ГДР.

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