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DIGITAL SIGNAL PROCESSING APPLICATION IN NUCLEAR SPECTROSCOPY

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Digital signal processing algorithms for nuclear particle spectroscopy are described along with a digital pile-up elimination method applicable to equidistantly sampled detector signals pre-processed by a charge-sensitive preamplifier. The signal processing algorithms provided as recursive one- or multi-step procedures which can be easily programmed using modern computer programming languages. The influence of the number of bits of the sampling analogue-to-digital converter to the final signal-to-noise ratio of the spectrometer considered. Algorithms for a digital shaping-filter amplifier, for a digital pile-up elimination scheme and for ballistic deficit correction were investigated using a high purity germanium detector. The pile-up elimination method was originally developed for fission fragment spectroscopy using a Frisch-grid back-to-back double ionisation chamber and was mainly intended for pile-up elimination in case of high alpha-radioactivity of the fissile target. The developed pile-up elimination method affects only the electronic noise generated by the preamplifier. Therefore, the influence of the pile-up elimination scheme on the final resolution of the spectrometer investigated in terms of the distance between piled-up pulses. The efficiency of developed algorithms compared with other signal processing schemes published in literature.

Keywords: x- and gamma-ray spectroscopy, computer data analysis, ionization chambers, interpolation; curve fitting, numerical differentiation and integration, integral and integrodifferential equations.

Introduction

The basic element of a nuclear spectrometer is a detector combined with a charge-sensitive preamplifier. The measurement of the kinetic energy of a radiation particle relies on the processing of the electric current pulse created by the motion of the free electrons/holes released during the ionization of the detector material. The step like pulse at the output of a charge-sensitive preamplifier (CSPA) is the result of an integration of the detector current. The height of the pulse is proportional to the total charge produced during the deceleration of the charged particle. This pulse height can be measured as the difference between the baseline (before the particle hits the detector) and the peaking value (after total charge collection was done) of the pulse after filtering out the useless high frequency components with the help of a shaping-filter amplifier (SFA). This principle is implemented in commercially available nuclear electronic modules performing signal processing which can be represented as a sequence of mathematical procedures applied to the waveform of a continuous signal. The output pulse of the SFA can have either a Gaussian or flat top pulse shape that can be used as the input to a peak-sensing analog-to-digital converter (ADC) for a pulse height analysis. From a mathematical point of view one can consider the signal evolution from the detector to the ADC as a sequence of transformations that can be described by precisely defined mathematical expressions. Recently, using waveform digitizer (WFD) the mathematical transformations implemented in analog electronic modules can be implemented software-wise using digital signal processing (DSP) algorithms. Some examples of the implementation of DSP techniques to nuclear particle spectroscopy are reported in Refs. [1 - 5], where the analogue pulse processing modules with continuous time signals were replaced by direct calculations with sampled signals – discrete values taken from a continuous signal at equidistantly separated points.

Hardware and software used in measurements

Detector pulses were digitized using a TDS3054B digital storage oscilloscope from Tektronix Inc. as shown in Fig. 1. The TDS3054B allowed signal digitization with accuracy of 8 bit and with frequency of up to $5 \cdot 10^9$ samples/s. Four waveforms of 10000 samples each can be simultaneously recorded in local memory of the oscilloscope, controlled by a remote PC via Ethernet connection. Data exchange between the oscilloscope and the PC was facilitated by the TekVISA [6] software library easily accessible from the Tektronix Inc. company web site. Both the data acquisition and the data analysis software were developed using Microsoft Visual C++ for Windows XP. Let us consider, for example, a high-purity germanium (HPGe) detector irradiated by a ^{60}Co calibration source. Detector pulses after being processed by a CSPA (with -3dB bandwidth of ~15 MHz) were digitized by the oscilloscope with 250 MHz and 8 bit (256 levels) accuracy. Let us consider Fig. 1 where the block diagram of the CSPA is presented to demonstrate the transformation of the detector signal. Let t being the time interval passed from the start of the measurement when the particle hits the detector and $I(t)$ is the instant value of the electric current flowing through the detector at time t. Assuming a particle entering the detector at $t = 0$ the following relation between the detector current and the preamplifier output voltage $V(t)$ is valid:

$$V(t) = \int_{0}^{\infty} I(\tau)h(t-\tau)d\tau . \qquad (1)$$

The measured output *V(t)* can be used directly for the determination of the total charge created by the particle assuming that the current pulse duration is short enough and *h(τ)~* const. Thanks to digitisation the solution of Eq. (1) with respect to *I(τ)* becomes possible allowing a direct evaluation of the detector current and the total charge. Inversion of Eq. (1) yields:

$$I(t) = \int_{0}^{\infty} V(\tau)H(t-\tau)d\tau . \qquad (2)$$

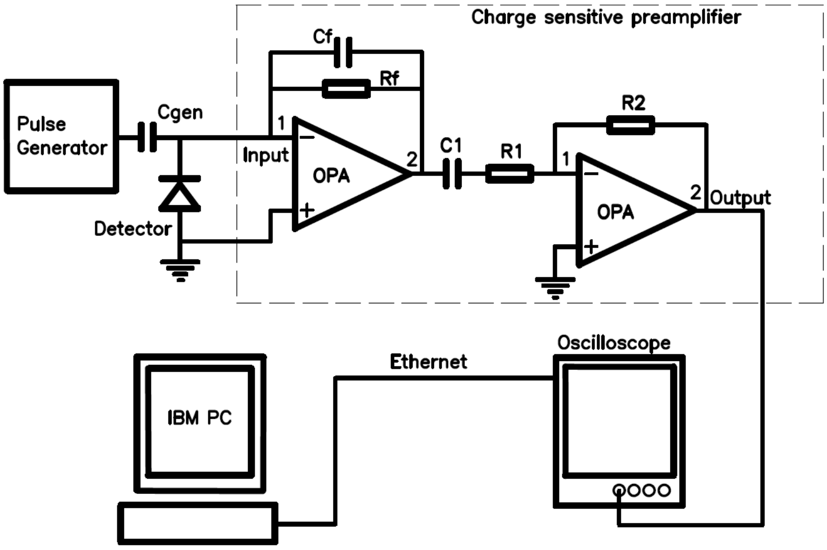

Fig. 1. Experimental setup.

Description of the differentiation and shaping algorithms

Eq. (1), providing the relation between the preamplifier output signal and the detector current stimulated by the ionising particle, can be used as a starting point for the determination particle's kinetic energy. In practice, the function $h(t-\tau)$ can be represented analytically as:

$$h(t-\tau) = \frac{1}{\alpha}\exp(-(t-\tau)/\alpha) \text{ for } t \geq \tau,$$

$$\text{and } h(t-\tau) = 0, \text{ for } t < \tau. \qquad (3)$$

The parameter α is the decay time of the preamplifier and is in the range of 50 - 100 μs. A δ-function like current pulse applied to a charge sensitive preamplifier input produces according to Eq. (1) a step-like output signal with fast rise time and exponential decay time α (see left part of Fig. 2). The height of the pulse is proportional to the total charge composing the current pulse, hence to the kinetic energy of the ionising particle. In the present measurement the output signal of the preamplifier was digitized and stored for further off-line DSP analysis. According to Shannon's theorem, sampled and continuous representations of the signal are equivalent to each other if the signal sampling was made properly [5, 7]. An original function *i(t)* for the detector current can be found from Eq. (1) using the following recursive expression representing a differentiation:

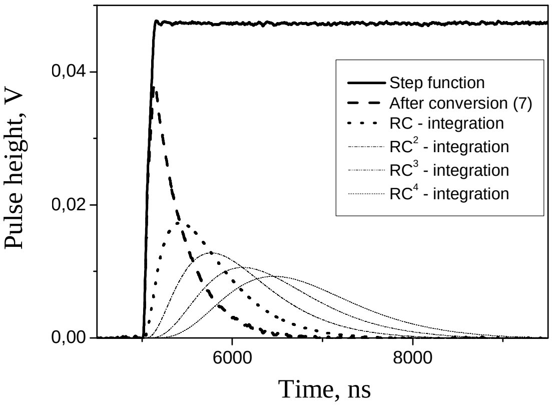

Fig. 2. Passage of a step like pulse trough the CR-RC^4 filter.

$$i_k = \lambda v_k - v_{k-1}, \; k = 0,1,2,\ldots N \qquad (4)$$

where $i_k = i(t_k)$, $v_k = v(t_k)$ are values taken at the sampling points t_k, $i_0 = 0$, and $\lambda = \exp(1/\alpha)$ is defined by the used preamplifier. Integrating this current signal over time according to expression:

$$Q(t)=\int\limits_{0}^{t} i(\vartheta)d\vartheta \qquad (5)$$

leads to the total charge flown through the detector as the peak value of the step pulse $Q(t)$.

In the optimal analog signal processing procedure [8] the signal *Q(t)* first passed the C-R (circuit consisting of a serially linked capacitance – C and a resistance – R) differentiator with the transfer function as follows:

$$Df(\tau)=\delta(\tau)-\frac{1}{A}\exp(-\frac{\tau}{A}) \qquad (6)$$

δ(τ) is Dirac's delta-function and A is a shaping constant. Usually differentiation is followed by 3 - 4 successive R-C integrations with the following transfer function

$$Int(\tau)=\frac{1}{A}\exp(-\frac{\tau}{A})\,. \qquad (7)$$

After differentiation, the step like function is transformed into an exponential decay function with the peak value proportional to the total charge value collected on the detector electrode. Successive integrations are needed to improve the signal-to-noise ratio resulting in an almost Gaussian shaped pulse. Historically, signal processing using Eqs. (6) and (7) was derived by optimisation of the SNR using signal filtering after differentiation [8]. The following equation provides the mathematical description of the transformations (6) and (7):

$$V^{Out}(t)=\int\limits_{0}^{t}\frac{dV^{In}(\tau)}{d\tau}W(t-\tau)d\tau=$$

$$=V^{In}(t)W(0)-\int\limits_{0}^{t}V^{In}(\tau)\frac{dW(t-\tau)}{d\tau}d\tau\,. \qquad (8)$$

For example, it can be easily verified that the convolution of $\frac{dV^{In}(t)}{dt}$ with the weighting function:

$$W(\tau)=\frac{1}{A}\exp(-\frac{\tau}{A}) \qquad (9)$$

is equivalent to the transformations given by Eqs. (6) and (7). The peak value of $V^{Out}(t)$, corresponding to the total measured charge of the ionizing particle, is proportional to the particle's kinetic energy. Changing the weighting function *W(t)* one can obtain different signal shape and signal processing schemes optimized for particular experiment, and may be chosen as a compromise between counting rate and resolution. One can verify the validity of the following transformation:

$$V^{In}(t)W(0)-\int\limits_{0}^{\infty}V^{In}(\tau)\frac{dW(t-\tau)}{d\tau}d\tau=$$

$$=V^{In}(t)-\int\limits_{0}^{\infty}V^{In}(\tau)\frac{dW(t-\tau)}{d\tau}d\tau\,. \qquad (10)$$

Using the substitution $V_k^{In}=V^{In}(k\Delta)$, $V_k^{Out}=$ $=V^{Out}(k\Delta)$ and $V_k^{Int}=\int\limits_{0}^{\infty}V^{In}(\tau)\frac{dW(k\Delta-\tau)}{d\tau}d\tau$ one can get the following relations from Eq. (16):

$$V_{k+1}^{Int}=V_k^{Int}\times A+V_k^{In},\ \ V_k^{Out}=V_k^{Int}-V_k^{In}\,. \qquad (11)$$

Applying N subsequent integrations using relation $V_{k+1}^{Int}=V_k^{Int}\times A+V_k^{In}$, where at each next step the output signal from the previous step is treated as the input signal for the next step is identical to passing of the signal through a CR-RC^n – filter. Fig. 2 illustrates how a step like pulse is transformed when passed through the CR-RC^4 filter. As an example a pulse height distribution for a ^{60}Co source measured using described above setup presented in Fig. 3 with the resolution was found to be 2.15 keV.

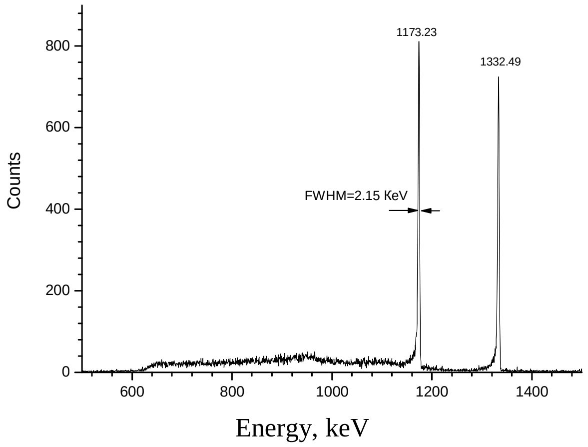

Fig. 3. Energy spectrum of ^{60}Co calibration source.

Investigation of pile-up elimination scheme

In practice complete pulse isolation can not be realised and as a result some pulses were distorted by pile-ups. According to the used data acquisition scheme, the pulse triggering the acquisition hardware was located at a fixed position of the waveform called trigger position. If an additional pulse was detected at the distance less than ±L from trigger, then the acquired waveform undergo pile-up elimination procedure illustrated in Fig. 4. The area

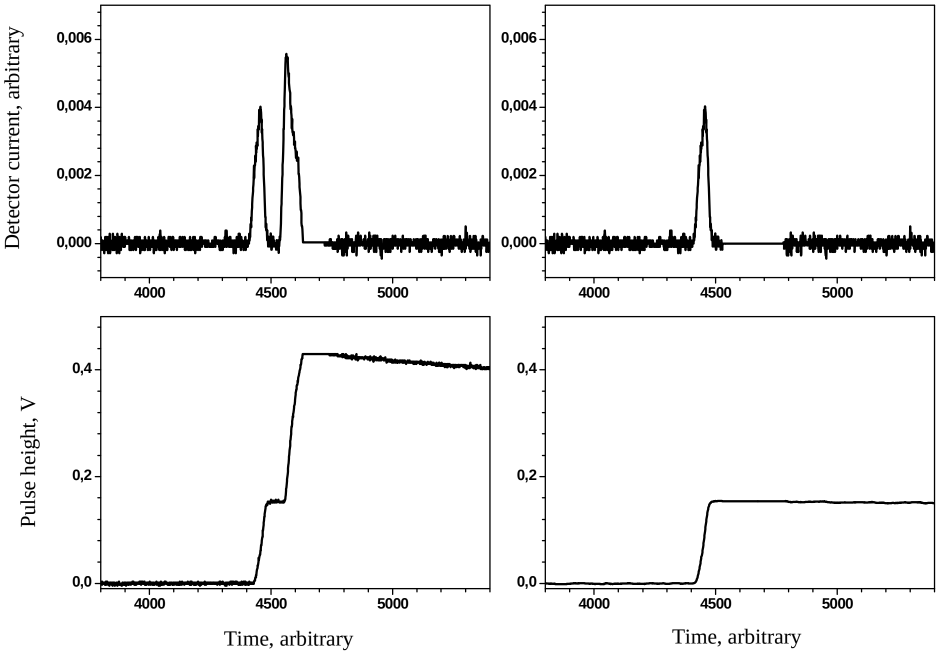

Fig. 4. Illustration of the pile-up elimination scheme.

Energy resolution for different SFA measured using the pulse generator and the source

Filter	Res. measured with high precision pulse generator, keV	Resolution of (1173.2 keV) line, keV
CR-RC^4	1.40	2.15 ("ballistic deficit" corrected)
Trapezoidal	1.40	2.20("ballistic deficit" corrected)
Ref. [9]	2.00	2.90 ("ballistic deficit" not corrected)

occupied by the detected pile-up pulse in the waveform (right hand part of Fig. 4) was forced to zero and then the target pulse height was calculated using pulse processing with the CR-RC^4 shaping. Obviously, the resolution of the spectrometer after implementing pile-up elimination scheme should be compared with the spectrometer resolution, when pile-up flagged waveforms are completely excluded from the analysis. In present work pile-up pulses were simulated by forcing of the average signal width time interval to zero and the resolution was compared with the resolution measured with undisturbed waveforms. The distance between the analyzed pulse and the simulated pile-up was fixed to a certain value and the entire acquired data set was analysed to determine the resolution for the 1.173 MeV line of the ^{60}Co source. The procedure was repeated with different distances between the pulse and the simulated pile-up and the dependence of the resolution on the distance was measured and plotted in Fig. 5 for the CR-RC^4 filter.

It should be noted that resolution degradation in the case of using described pile-up elimination scheme depends on the distance between pulses and at the minimum distance (when pulses still can be treated as separate ones) it is ~3 times lower than for isolated pulse. Such degradation however, can be controlled by the software and, therefore experimentalist can make a choice between the counting rate and the resolution in measurement.

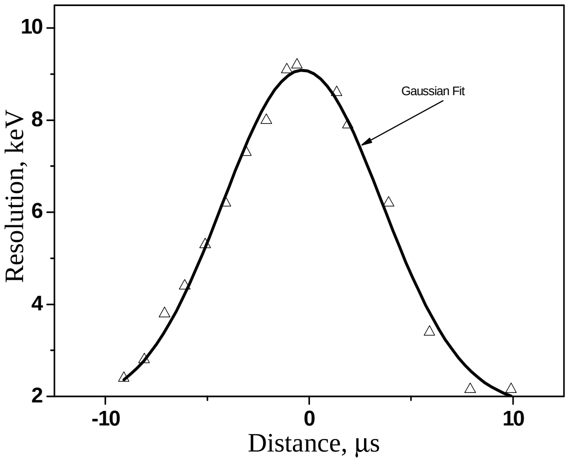

Fig. 5. Resolution as a function of the distance between pile-ups.

The statistical accuracy of the output signal measurement improved in proportion to the square root of the number of samples of output signal involved in calculation, which is proportional to the width of the weighting function *w(t)*. With the sampling frequency of 250 MHz and with the width of the weighting function of ~10 μs that was used in data analysis with RC-CR^n – SFA in the reported work ~2500 samples of input function was utilised for calculation of one sample of the output function gives the improvement of SNR by factor of 50. The authors of Ref. [3] called this improvement as "bit gain factor" comparing the WFD based spectrometer with conventional peak sense ADC based analog spectrometer. Following that approach one can find that spectrometer used in this work had the same SNR as 11.84 bit conventional peak sense ADC.

Conclusions

Signal processing algorithms developed in this work were provided as recursive computational procedures that can be effectively used for computation. Comparison of developed algorithms with that described in literature showed almost 40 % improvement of the resolution in high resolution gamma-spectroscopy. From the sampled waveform of a detector signal amplified by a charge-sensitive preamplifier the detector current signal was first reconstructed and was used for pile-up elimination and true ballistic deficit correction of the detector charge. The pile-up elimination method was found to be very effective for fission fragment spectroscopy, although not demonstrated in the present paper. The basics of the signal sampling theory were briefly reviewed to demonstrate the method of calculation using sampled signal representation in the analysis procedure where the signal values are to be reconstructed between sample points. Influence of the number of bits of the sampling analogue-to-digital converter and the weighting function of the SFA to the final signal-to-noise ratio of the spectrometer considered and expressed as "bit gain factor" introduced in Ref. [3].

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АЛГОРИТМИ ЦИФРОВОЇ ОБРОБКИ СИГНАЛІВ ПРИ СПЕКТРОСКОПІЇ ЯДЕРНИХ ЧАСТИНОК

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Розглянуто алгоритми цифрової обробки сигналів при спектроскопії ядерних частинок і метод вилучення накладань імпульсів, дискретизованих з фіксованою частотою. Ці алгоритми сформульовано у вигляді рекурсивних процедур, зручних для програмування на сучасних алгоритмічних мовах. Досліджено вплив числа біт амплітудно-цифрового перетворювача на величину відношення сигнал/шум. Розроблено та досліджено алгоритми цифрового спектрометричного підсилювача з трапецеїдальним та CR-RC$_n$ формуванням, пристрої вилучення накладань та процедура корекції “балістичного дефіциту”. Випробування спектрометричних характеристик проведено експериментально з використанням детектора гамма-квантів з надчистого германію. Спершу вказані алгоритми розроблялися для спектроскопії осколків поділу ядер, але найбільш повне дослідження їх характеристик стало можливим при застосуванні детектора високої роздільної здатності. Досліджено вплив методу вилучення накладань на роздільну здатність спектрометра залежно від відстані між досліджуваними імпульсами.

Ключові слова: детектори випромінювання, гамма-спектроскопія, ядерний розпад, програмне забезпечення, швидкий оцифровувач, імпульс, числове диференціювання та інтегрування, інтегральні та інтегро-диференційні рівняння, інтерполяція, іонізаційні камери.

АЛГОРИТМЫ ЦИФРОВОЙ ОБРАБОТКИ СИГНАЛОВ ПРИ СПЕКТРОМЕТРИИ ЯДЕРНЫХ ЧАСТИЦ

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Рассмотрены алгоритмы цифровой обработки сигналов при спектрометрии ядерных частиц и метод исключения наложений импульсов, дискретизованных с фиксированной частотой. Эти алгоритмы сформулированы в виде рекурсивных процедур, удобных для программирования с использованием современных алгоритмических языков. Исследовано влияние числа бит амлитудно-цифрового преобразователя на величину отношения сигнал/шум спектрометра. Разработаны и испытаны алгоритмы цифрового спектрометрического усилителя с трапециоидальным и CR-RC$_n$ формированием, устройства исключения наложений и процедура коррекции "баллистического дефицита". Испытания спектрометрических характеристик проведены экспериментально с использованием детектора гамма-квантов из сверхчистого германия. Изначально указанные алгоритмы разрабатывались для спектроскопии осколков деления ядер, но наиболее полное исследование их характеристик оказалось возможным с применением детектора высокого разрешения. Исследовано влияние метода исключения наложений на разрешение спектрометра в зависимости от расстояния между исследуемыми импульсами.

Ключевые слова: детекторы излучения, гамма-спектроскопия, ядерный распад, программное обеспечение, быстрый оцифровщик, импульс, численное дифференцирование и интегрирование, интегральные и интегро-дифференциальные уравнения, интерполяция, ионизационные камеры.

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