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S-METHOD FOR EVENT LOCALIZATION IN ANGER TYPE GAMMA CAMERA

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New S-method for gamma-quantum coordinate estimation in Anger type tomography gamma-camera is briefly described. S-method uses special metric and provides for gamma-camera large field of view. Comparison with well known and widely used methods for event localization like Anger approach and maximal likelihood (ML) and allied approaches is made and some results are shown.

Keywords: tomography, gamma-camera, event positioning, method.

Introduction

In such medical imaging techniques as single photon emission computed tomography (SPECT) and positron emission tomography (PET) an Anger type scintillation gamma camera is widely used as the imaging device. In scintillation camera a γ-ray photon interacts with the crystal (initial event) and then the signals V_i from an array of photomultiplier tubes (PMTs) are processed to provide information about the position of event [1].

Routine Anger approach (centroid type calculation) [2] for event localization gives the answer in explicit form, amount of computations as small as possible, but estimation is biased, especially near the edge of detector. As usual maximal likelihood (ML) and allied approaches [3, 4] gives good results and have good features (unbiased, robustness), but they require large computations. So to implement these approaches some simplifications are used.

In proposed S-method (or Shift-method) only a bit more computations are needed than for simplest Anger approach. S-method is bias-free and stable to variations of event energy. Besides it is convenient for implementation.

Typical PMTs layout in gamma-camera

Let us consider typical array of 59 PMTs hexagonal disposed as shown in Fig. 1.

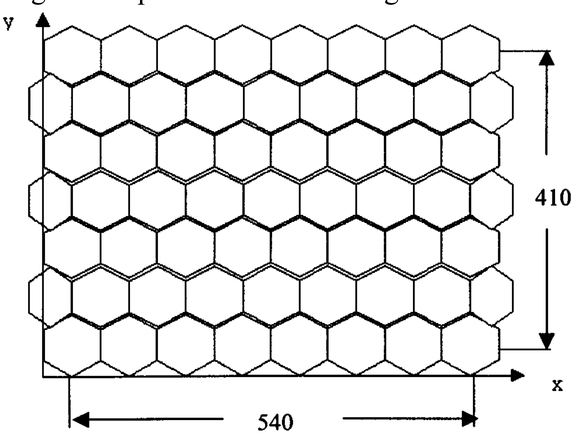


Fig. 1. Typical PMTs layout.

Crystal size is 590 × 470 mm. Centers of corner PMTs make up rectangle with size 540 × 410 mm. Usually this rectangle covers Field of View (FOV). As a rule only the signals from the group of PMTs nearest to event are used to estimate the single event position. Such group may consist of from 7 to 19 PMTs (main PMT and 6 PMTs in first ring, and 12 PMTs in second ring) for event in central zone of FOV.

Input data for one single event handling

Gamma-ray photon produces the scintillation. Thus, we have a group of PMTs which signals exceed a threshold. So, input data are:

1. $\mathbf{P_k} = (X_k, Y_k)$, k = 0,1, …, K (PMT positions (coordinates of the centres)).
2. V_k, k = 0,1, …, K (PMT output signals).

Let the largest signal PMT ("main" PMT) has number 0, i.e. its position is P_0, and its signal is V_0.

Light response function of PMT

Light response function (LRF) $F(\mathbf{r}) \equiv F(r, \theta)$ is individual characteristic for each PMT in a detecting system. Function $F(r, \theta)$ represents a dependence between PMT output signal and distant and direction from centre of PMT to event position. Function $F(r, \theta)$ is normalized, i.e. the unit energy of event is supposed. (Typical shapes of LRF and reverse LRF are shown in Figs. 2 and 3). For event energy E output signal will be $V = E \cdot F(\mathbf{r})$. $(r = F^{-1}(V/E))$.

To simplify the description we will consider LRF dependent only on distant r. Moreover, we assumed all LRF's are the same ($F_k(r) = F(r)$).

Measure for point remoteness from event position

Let the event with unit energy occur in unknown position $\mathbf{P_e} = (X_e, Y_e)$ and the obtained output PMT signals are V_k, k = 0,1, …, K. Let us consider test position $\mathbf{P_t} = (X_t, Y_t)$. For event with unit energy occur in position P_t, the PMT signals may be calculated as $W_i = F_i(\mathbf{P_t})$.

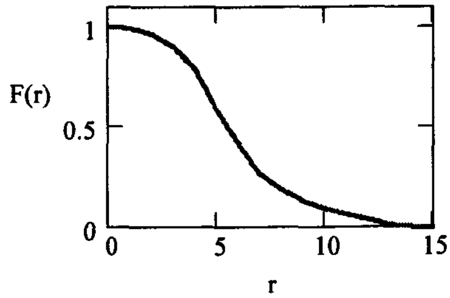


Fig. 2. Normalized LRF.

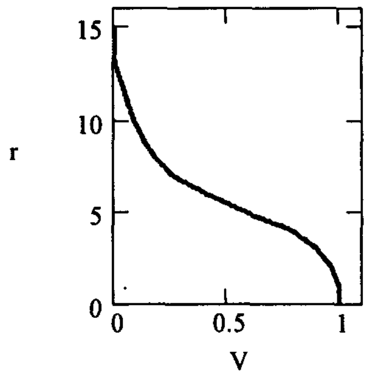


Fig. 3. Reversed LRF.

As P_e is unknown, instead of simple distance $\rho^2(\mathbf{P_t}, \mathbf{P_e}) = ||\mathbf{P_t} - \mathbf{P_e}||^2$, another measure on the base of **V** and **W** is used, for example

$$R = \sum(W_i - V_i)^2 = ||\mathbf{W} - \mathbf{V}||^2. \quad (1)$$

Since **W** depends on $\mathbf{P_t}$ expression (1) may be written in form:

$$R(\mathbf{P_t}) = \sum(W_i(\mathbf{P_t}) - V_i)^2 = ||\mathbf{W}(\mathbf{P_t}) - \mathbf{V}||^2. \quad (2)$$

Methods based on functional minimization

From theoretical point of view the maximal likelihood (ML) approach is statistically valid (if assumptions are true). Some other approaches like Chi square error (CSE) or Mean square error (MSE) are deduced from ML under some conditions.

In essence, ML and allied approaches aims at minimization of difference between **W**(P) and **V.** The approaches are distinguished by measure they explore and by method of minimization. For example, for LS (least squares) approach we have a typical situation. Measure used is (2). This measure depends on point $\mathbf{P_t}$ and minimum is reached when $\mathbf{P_t} = \mathbf{P_e}$. The estimation for event position usually is obtained by solving functional minimization problem:

$$\mathbf{P_e} = \text{Arg min } ||\mathbf{W}(\mathbf{P_t}) - \mathbf{V}||^2. \quad (3)$$

Anger approach

Even Anger approach – weighted centroid (WC), can be deduced from ML if we roughly approximate LRF (light response function) by triangular distribution function. The Anger method gives the answer in explicit form:

$$X = S \cdot \sum V_i \cdot X_i, \qquad Y = S \cdot \sum V_i \cdot Y_i, \quad (4)$$

where $S = 1/\sum V_i$, (X_i, Y_i) – coordinates of *i*-th PMT centre; V_i – output of *i*-th PMT.

In Anger approach amount of computations are as small, as possible (~120 arithmetic operations per event), but estimations are biased, especially near the edge of FOV. For this reason usually linearity correction is needed [5]. Some propositions to improve Anger approach leads to increasing of computations [6].

Shift-method

Let event with unit energy is detected. From SRF attribute "the signal is more – the distance is less" we conclude that event is somewhere in main PMT area. It is the first, rough information on position of event. Naturally, therefore, to choose $\mathbf{P_0}$ (the position of main PMT) as initial approximation for event position $\mathbf{P_e}$. Let consider $\mathbf{P_0}$ as test point, i.e. $\mathbf{P_t} = \mathbf{P_0}$. On the basis of a difference between W_i and V_i we should define in what direction and on what distance it is necessary to shift from $\mathbf{P_0}$ to hit the mark – unknown position $\mathbf{P_e}$. This shift-vector depends on **V** and **W**, and also on PMT coordinates **P**:

$$\mathbf{H} = \mathbf{H}(\mathbf{V}, \mathbf{W}, \mathbf{P}). \quad (5)$$

The event position we estimate as

$$\mathbf{P_e} = \mathbf{P}_0 + \mathbf{H}. \quad (6)$$

To define a size and a direction of shift **H** we will exploit a "pointing" vector **D** that we construct on the basis of the data (**V**, **W**, **P**):

$$\mathbf{D} = \sum V_i \cdot \mathbf{P}_i - \sum W_i \cdot \mathbf{P}_i - \mathbf{P}_0 \cdot (\sum V_i - \sum W_i). \quad (7)$$

Vector **D** not only serves as a measure of difference between the centre of main PMT and event position, but also specifies a direction to event.

In contrast to traditional approaches (ML and allied approaches) in S-method a measure of difference between W (P) and V is not functional, but vector.

Let's determine separately length and direction of **D**:

$$m = ||\mathbf{D}||, \quad \mathbf{u} = \mathbf{D}/m. \quad (8)$$

Pointing vector **D** is directed from point $\mathbf{P_0}$ to event, and its size is proportional to distance from $\mathbf{P_0}$ to event position. Simplifying a picture it is possible to suppose that there is the dependence between distance to event and module of vector **D**:

$$r = r(m). \quad (9)$$

After calculating vector **D**, we determine size r and direction **u** of the shift, and then get vector **H** as

$$\mathbf{H} = r\,(m) \cdot \mathbf{u}. \qquad (10)$$

So, the idea of S-method for event position estimation is following: On the basis of the data (V, W, P) we have to calculate pointing vector D, then, using a tabulated dependence r = r (m) to determine vector H and, finally, to shift from the centre of main PMT by H:

$$\mathbf{P}_e = \mathbf{P}_0 + r\,(m) \cdot \mathbf{u}. \qquad (11)$$

As initial approximation $\mathbf{P}_0$ is defined in advance, part of calculations may be done preliminary and stored in look-up tables. Only a bit more computations are needed than for Anger approach. A lot of issues are not considered in this description. Name some of them:

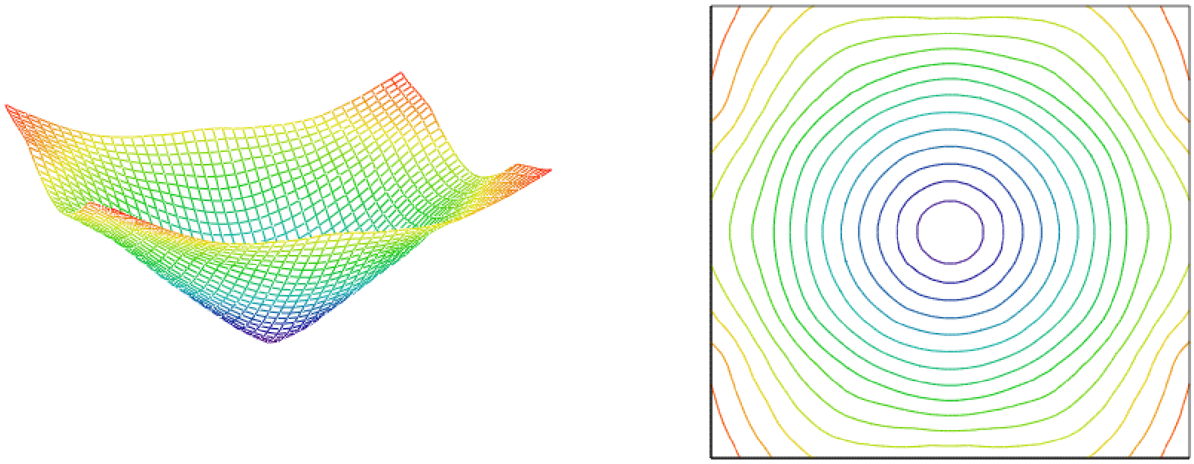
1. The event energy is not unit.
2. SRF depends on θ and on k.
3. Vector **u** is not accurate direction for the shift (in (10 - 11) corrected **u** is used).
4. Transition from one PMT area to another must be smoothed.

All of these issues were successfully overcome in S-method. (The solutions of these problems are technical details of S-method). The S-method may be considered as special iterative method, but already first step of iterations gives good approximation for event position.

Some comparisons

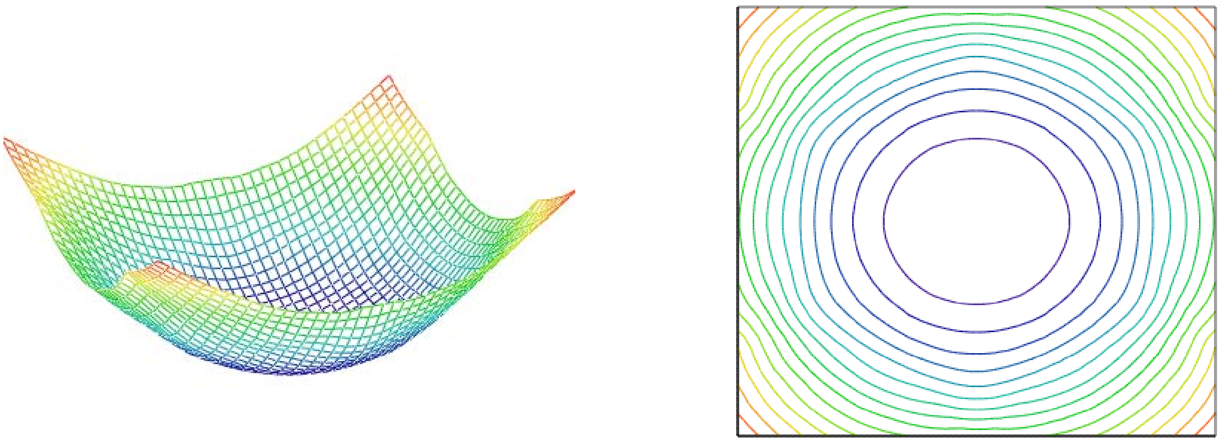
In the Figures below some comparative calculations that illustrate the behavior of measure (2) which is used in LS and size of pointing vector which is used as a measure of a difference in the S-Method are shown. Signals **V** and **W** were calculated for point in centre of main PMT and for points in grid covered main PMT area. It is clear from Figs. 4 and 5 that searching for a minimum of (2) is more difficult.

The minimum of size of pointing vector is well expressed both at edge and in a corner of FOV as Figs. 6 - 9 shown.



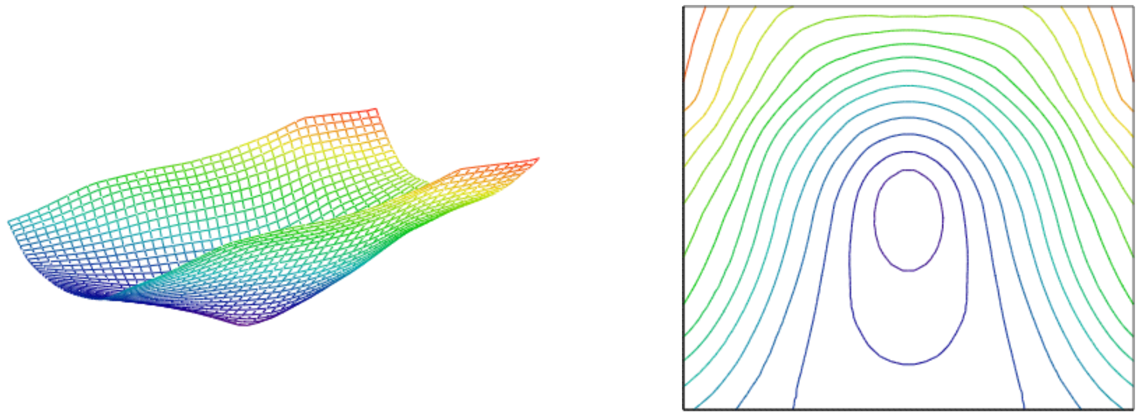
a *b*

Fig. 4. Size of pointing vector as surface (*a*) and contour plot (*b*) when the main PMT is in FOV central zone.



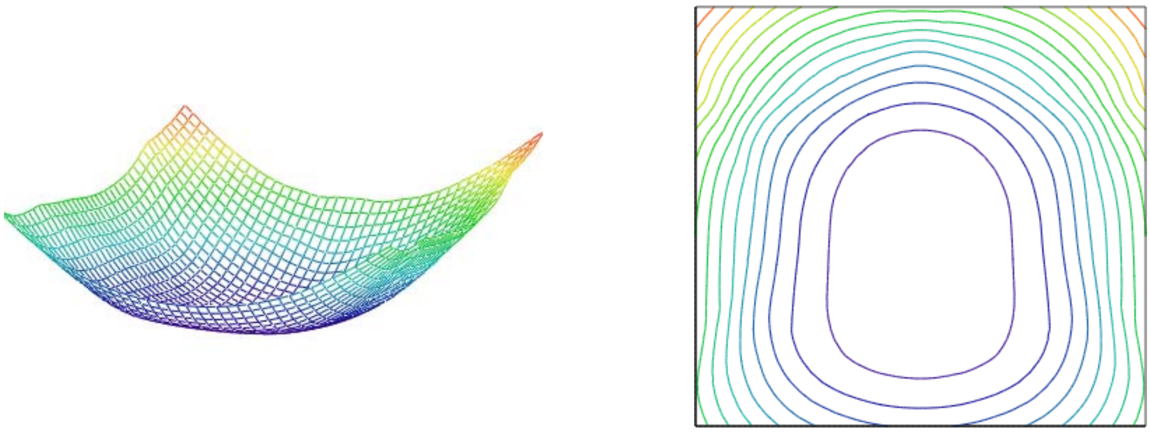
a *b*

Fig. 5. Measure (2) for **W – V** as surface (*a*) and contour plot (*b*) when the main PMT is in FOV central zone.



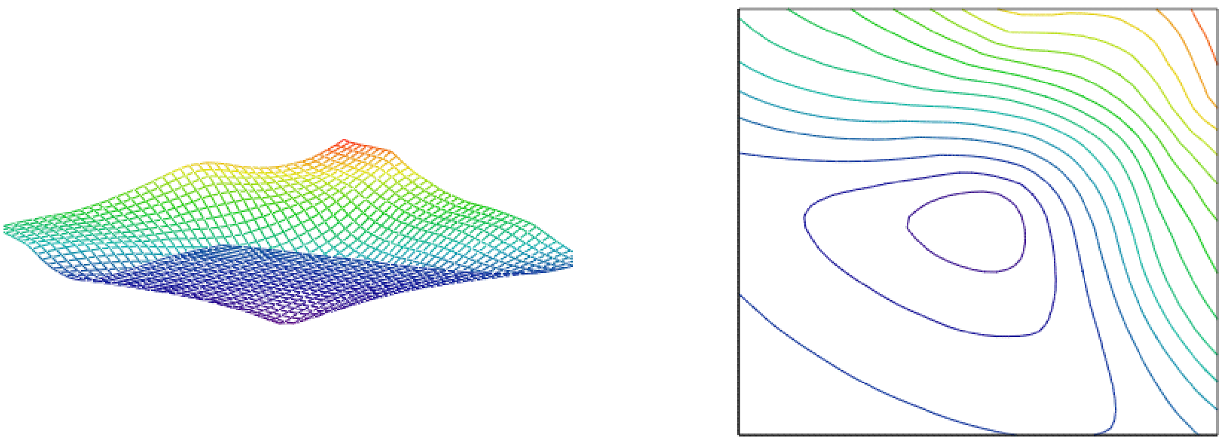
a *b*

Fig. 6. Size of pointing vector as surface (*a*) and contour plot (*b*) when the main PMT is at the edge of FOV.



a *b*

Fig. 7. Measure (2) for **W – V** as surface (*a*) and contour plot (*b*) when the main PMT is at the edge of FOV.

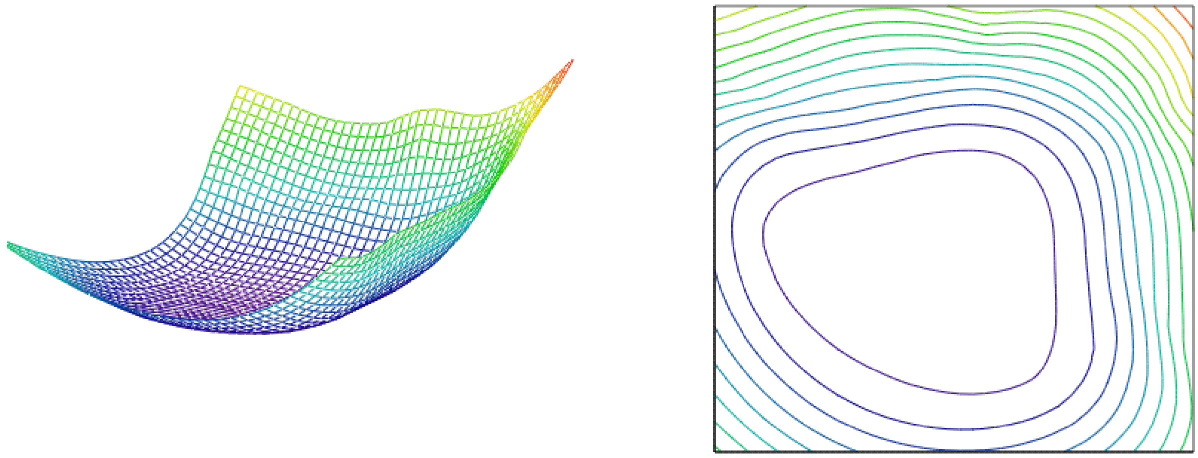


a *b*

Fig. 8. Size of pointing vector as surface (*a*) and contour plot (*b*) when the main PMT is in the corner of FOV.

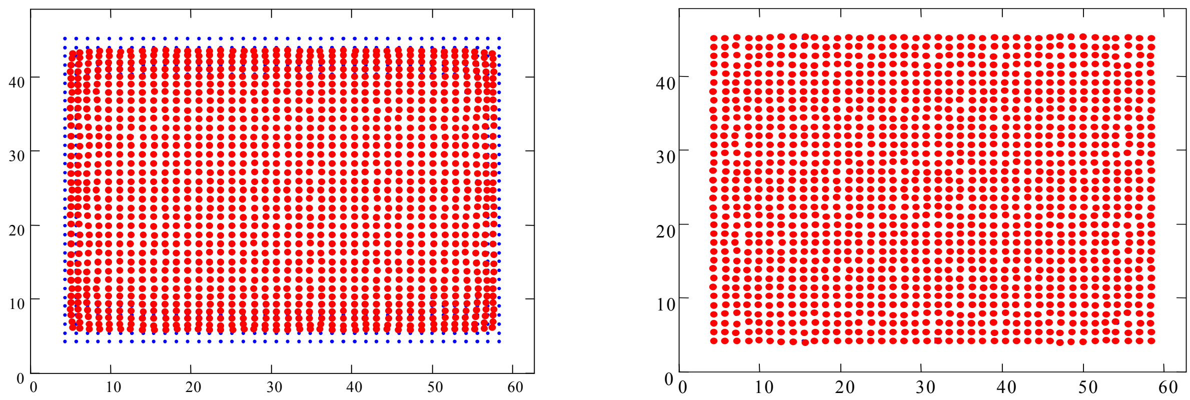
Also the comparison between Anger approach and S-method was made by modeling computations for regular grid of points in the area of FOV. For tradition Anger approach we have noticeable non-linearity and significant bias for points near edge (Fig. 10, *a*)). The S-method for mentioned grid of points gives result with large FOV and with acceptable linearity (not worse than that for Anger method) (Fig. 10, *b*)).

The procedure of linearity correction [5] also can be used with S-method and for large FOV at that. So, Anger type gamma-camera with PMTs array like in Fig. 1 can have FOV 55 × 42 cm instead of today's 53 × 39 cm due to new S-method.



a *b*

Fig. 9. Measure (2) for **W – V** as surface (*a*) and contour plot (*b*) when the main PMT is in the corner of FOV.



a *b*

Fig. 10. Estimation of point positions in Anger approach (*a*) and in S-method (*b*).
(Points of grid are marked as dot (·) and estimated positions as circle (•).)

REFERENCES

1. *Mathematics and Physics of Emerging Biomedical Imaging* // National Research Council. - Washington, D.C.: National Academy Press, 1996. - 242 p.
2. *Anger H. O.* Scintillation camera // Rev. Sci. Instrum. - 1958. - Vol. 29. - P. 27 - 33.
3. *Joung J., Miyaoka R. S., Kohlmyer S., Lewellen T. K.* Implementation of ML based positioning algorithms for scintillation cameras // IEEE Trans. Nucl. Sci. - 2000. - Vol. 47. - P. 1104 - 1111.
4. *Joung J., Miyaoka R. S., Kohlmyer S., Lewellen T. K.* Investigation of bias-free positioning estimators for the scintillation cameras // IEEE Trans. Nucl. Sci. - 2001. - Vol. 48. - P. 715 - 719.
5. *Sokolov A. M.* Linearity correction of two-dimension image in tomography gamma-camera // Nuclear Physics and Atomic Energy. - 2008. - No. 1(23). - P. 96 - 98.
6. *Vesel J., Petrillo M.* Improved gamma camera performance using event positioning method based on distance dependent weighting // Nuclear Science Symposium Conference Record, IEEE Iss. - 2005. - Vol. 5. - P. 2445 - 2448.

S-МЕТОД ОЦІНКИ КООРДИНАТ ПОДІЙ, ЩО РЕЄСТРУЮТЬСЯ ГАММА-КАМЕРОЮ АНГЕРА

О. М. Соколов

Дано короткий опис нового S-методу для оцінки координат подій, що реєструються детектором томографічної гамма-камери. S-метод застосовує спеціальну метрику та забезпечує гамма-камері велике поле бачення. Зроблено деякі порівняння з методами, що добре відомі та широко застосовуються для оцінки координат подій, такими як підхід Ангера, метод максимальної правдоподібності та ін., і показано результати порівняння.

Ключові слова: томографія, гамма-камера, подія, локалізація, метод.

S-МЕТОД ОЦЕНКИ КООРДИНАТ СОБЫТИЙ, РЕГИСТРИРУЕМЫХ ГАММА-КАМЕРОЙ АНГЕРА

А. М. Соколов

Приводится короткое описание нового S-метода для оценки координат событий, которые регистрируются детектором томографической гамма-камеры. S-метод использует специальную метрику и обеспечивает гамма-камере большое поле видения. Проведено сравнение с такими хорошо известными и широко применяемыми для оценки координат событий методами, такими как подход Ангера, метод максимума правдоподобия и др., и показаны результаты сравнения.

Ключевые слова: томография, гамма-камера, событие, локализация, метод

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