

**THE FUSION OF HEAVY IONS WITHIN THE TWO STEP REACTION MODEL**

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We consider the approaching stage of fusion-fission reactions. The state of the total system is characterised by the three collective parameters: the quadrupole deformations of the target nucleus and the incoming ion and the distance between their centers of mass. We assume “nose to nose” orientation of the projectile and target. The dynamics of the process is described by the Langevin equations for the mentioned above collective parameters. The shell effects of the projectile and target are taken into account in the interaction energy. The numerical calculations are carried out for two reactions: reactions with deformed (<sup>100</sup>Mo + <sup>100</sup>Mo → <sup>200</sup>Po) and spherical (<sup>208</sup>Pb + <sup>18</sup>O → <sup>226</sup>Th) in ground state ions. It is shown that the account of shell effects in the deformation energy influences not only the fusion process as a whole (the fusion barrier height, the dependence of the touching probability on the initial kinetic energy) but also the processes in each ion (their shape and excitation energy).

*Keywords:* fusion-fission reactions, Langevin equations, deformation energy, shell effects.

**Introduction**

The investigation of the fusion-fission reactions is one of current problem of nuclear physics. The superheavy elements are produced by the fusion-fission reactions. The theoretical description of fusion-fission reactions aims at the reproduction of the existing experimental data on the production of superheavy elements and at the prediction of optimal combinations of target and projectile for the synthesis of new elements.

The fusion-fission reactions are often described within a stochastic approach, by the Focker - Planck [1], or Langevin [2 - 4] equations. Here we present a further development of the model suggested in [4]. The description of fusion-fission reaction in this model is carried out in two steps: the approaching of ions up the touching point and the evolution of the compact system. We have improved the description of the first stage. In [4] the potential energy of ions was determined within the liquid drop model, the

mass parameter was calculated by the Werner - Wheeler method [5] and the friction parameter – within the one body dissipation model [6]. The colliding nuclei were assumed to be spherical at the initial moment. This was a good approximation for the magic nuclei which are spherical in the ground state but not so good for the nuclei deformed in the ground state. In the present work we take into account the shell effects in the deformation energy of colliding nuclei, what cause the deviation from the spherical symmetry. Thus, the model would correspond more closely to the experimental situation.

**The dynamical model for the approaching stage**

We consider here the approaching phase of two heavy ions. The distance between their centers of mass is denoted by  $r$ . The shape of the surface of ions is described in terms of Cassini ovaloids, which are defined in parametric way as [7]:

$$\begin{aligned} \rho_s(x) &= \frac{1}{\sqrt{2}} \left[ (R_0^4 + 2sR_0^2(2x^2 - 1) + s^2)^{1/2} - R_0^2(2x^2 - 1) - s \right]^{1/2}, \\ z(x) &= \frac{\text{sign}(x)}{\sqrt{2}} \left[ (R_0^4 + 2sR_0^2(2x^2 - 1) + s^2)^{1/2} + R_0^2(2x^2 - 1) + s \right]^{1/2}. \end{aligned} \tag{1}$$

Here  $\rho_s$  and  $z$  are the cylindrical coordinates of the point on the nuclear surface and  $x$  is a parameter given on the interval [-1, 1]. The  $R_0$  is the radius of spherical nuclei with the same volume. The parameters  $\alpha \equiv s / R_0^2$  fixes the deformation of the target ( $\alpha_t$ ) or projectile ( $\alpha_p$ ). These are the two addi-

tional collective variables which specify the state of the system. In calculations we assumed that the target and projectile are oriented “nose to nose” (their symmetry axes coincide).

The dynamics of the collision is described by the Langevin equations for  $r, \alpha_t, \alpha_p$  variables,

$$\begin{aligned} \frac{dr}{dt} &= p/m, & \frac{dp}{dt} &= -\left[ \frac{dV_{pot}(r, \vec{\alpha})}{dr} + K_r^r \frac{p}{m} + \sum_j K_r^j \frac{\pi_j}{D_j} \right] + \xi_r(t), \\ \frac{d\alpha_i}{dt} &= \frac{\pi_i}{D_i}, & \frac{d\pi_i}{dt} &= -\left[ \frac{dV_{pot}(r, \vec{\alpha})}{d\alpha_i} + K_i^r \frac{p}{m} + \sum_j K_i^j \frac{\pi_j}{D_j} \right] + \xi_i(t). \end{aligned} \quad (2)$$

In Eq. (2)  $i$  attains the value  $t$  (target) or  $p$  (projectile),  $\vec{\alpha} = \{\alpha_p, \alpha_t\}$ ,  $m$  is their reduced mass. The  $\xi_r(t)$  and  $\xi_i(t)$  are normal distributed random forces. The  $V_{pot}$  is the potential energy of the target and projectile. The potential energy  $V_{pot}$  includes the energies of Coulomb  $V_{Coul}$  and nuclear  $V_{GK}$  interaction [8], the rotational energy  $V_{rot} = \hbar^2 L^2 / 2(mr^2 + J_{pro} + J_{tar})$  with  $J_{pro}$  and  $J_{tar}$  being the rigid-body moments of inertia of the projectile and target, and the deformation energy of the projectile and target

$$V_{pot} = V_{Coul} + V_{GK} + V_{rot} + E_{def}, \quad E_{def} = E_{def}^{LDM} + E_{shell}. \quad (3)$$

The deformation energy was calculated within macroscopic-microscopic approach [9, 10] as the sum of the liquid drop deformation energy plus the shell correction. The dependence of the shell correction to the excitation energy (temperature) was parameterized in the form  $E_{shell} = E_{shell}(T=0)e^{-\gamma T^2 a}$ . The level density parameter  $a$  and the damping parameter  $\gamma$  were taken the same as in [11, 12]. The friction and mass parameter for the  $r$ -motion were defined in the same way as in [3]:

$$K_i^r = \frac{1}{2} K_r^r \frac{R_{i0}}{\sqrt{\alpha_i + 1}}, \quad K_j^i = \frac{1}{4} K_r^r \frac{R_{i0} R_{j0}}{\sqrt{(\alpha_i + 1)(\alpha_j + 1)}} + \delta_{ij} K_i.$$

Here the mass parameters  $D_i$  and the friction coefficients  $K_i$  related to the variation of the shape of the target and projectile were calculated within the linear response theory and locally harmonic approximation [13, 14].

The equations (2) were integrated numerically starting from the initial value  $r = r_{in}$  until the ions would touch at the touching point  $R_{touch}$  or, after reaching some minimal value (larger than  $R_{touch}$ ),  $r$  would start to increase and reach  $r_{in}$  again. As it was shown earlier, due to the random force some ions can not reach the touching point even if they overcame the fusion barrier. The value of  $R_{touch}$  is defined by the relation  $R_{touch} = R_1 + R_2 + (a_{d,t} + a_{d,p})/2$  [4], where  $2R_1$  and  $2R_2$  are the dimensions of the

(deformed) ions in the  $r$ -direction, and  $a_{d,t}$ ,  $a_{d,p}$  are the diffuseness of their density distribution. We have checked that the increase of  $r_{in}$  does not change the results of calculations but increases substantially the computation time. Besides  $r_{in}$  at the initial moment the kinetic energy of incoming ions and the angular momentum  $L$  are fixed.

Due to the action of random forces, we get not the strictly defined quantities at the touching point but the distributions. The calculations have to be repeated many times until the results become stable with respect to the number of trajectories. By trajectory we mean here the dependence of  $r$  on time for a given calculation.

On each integration step we calculate the dissipated energy by the kinetic energy loss,

$$E_{dis} = E_{cm} - \frac{p_r^2}{2m} - \sum_i \frac{\pi_i^2}{2D_i} - V_{pot}, \quad (4)$$

where  $E_{cm}$  is the center-of-mass energy. The dissipated energy defines the temperature (excitation energy) of the system at the touching point.

The touching probability  $T_L$  is calculated as the ratio of the number of trajectories, which have reached the touching point  $N_{touch}$ , to the total number  $N_L$  of considered trajectories  $T_L = N_{touch} / N_L$ . Besides the touching probability by solving equations (2) we find the potential energy of the system, the excitation energy and the deformation of the target and projectile, at the touching point. These data are used as the initial conditions for the description of evolution of the compact system.

### The results of numerical calculations

The dependence of the deformation energy of  $^{208}\text{Pb}$ ,  $^{100}\text{Mo}$  and  $^{18}\text{O}$  on the deformation parameter  $\alpha$  is shown in Fig. 1. One can see that due to the shell effects the deformation energy of  $^{208}\text{Pb}$  and  $^{18}\text{O}$  gets stiffer. Consequently, the deformation of  $^{208}\text{Pb}$  and  $^{18}\text{O}$  is almost unchanged during the approaching of the ions. In case of  $^{100}\text{Mo}$  the shell effects lead to the appearance of two minima in the deformation energy. I.e.,  $^{100}\text{Mo}$  can be oblate or prolate in the ground state. These both possibilities should be taken into account by the formulation of initial conditions for the system (2).

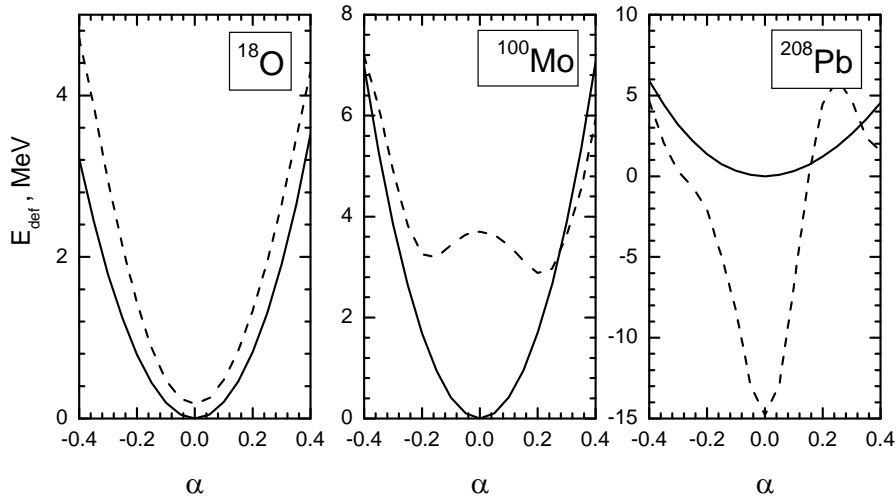


Fig. 1. The total (dash) and the liquid drop (solid) deformation energy (MeV) for  $^{18}\text{O}$ ,  $^{100}\text{Mo}$  and  $^{208}\text{Pb}$  as function of the deformation parameter  $\alpha$ .

The Fig. 2 shows the dependence of the mass parameter  $D_i(\alpha)$  and reduced friction coefficient  $K_i(\alpha)/D_i(\alpha)$  on the deformation and temperature  $T$ . One can see the strong variation of  $D_i(\alpha)$  with temperature. At the small temperature the main con-

tribution to  $D_i(\alpha)$  comes from the pairing correlations. This contribution disappears above critical temperature  $T_{\text{crit}}$  at which the pairing gap turns into zero.

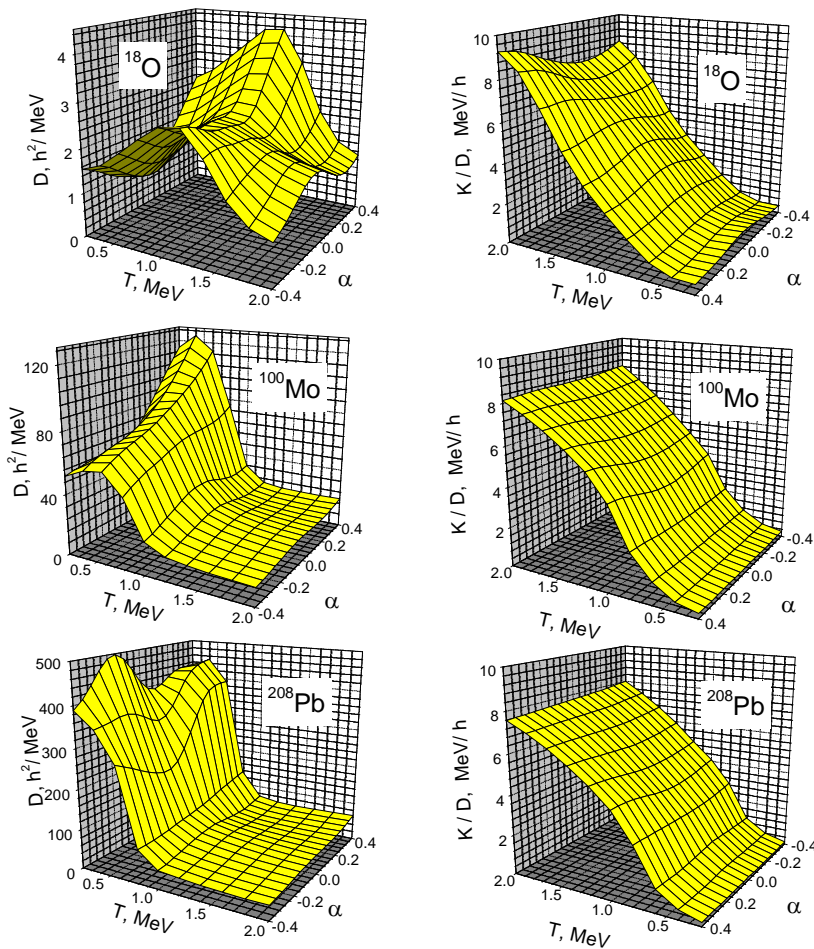


Fig. 2. The dependence of the mass parameter  $D_i(\alpha)$  (left) and the reduced friction coefficient  $K_i(\alpha)/D_i(\alpha)$  (right) on the deformation parameter  $\alpha$  and the temperature  $T$  for the same nuclei as in Fig. 1.

It can be seen from the structure of Langevin equations (2) that their solutions depend essentially not on the friction  $K_i(\alpha)$  or mass parameters  $D_i(\alpha)$  separately, but only on the ratio  $K_i(\alpha)/D_i(\alpha)$ . The temperature dependence of  $K_i(\alpha)/D_i(\alpha)$  is very different in the macroscopic and microscopic approaches. The ratio of wall friction over inertia of irrotational flow is practically constant. While the microscopic approach shows a fast increase of  $K_i(\alpha)/D_i(\alpha)$  with temperature.

At small temperatures the ratio  $K_i(\alpha)/D_i(\alpha)$  is very small. It means that the loss of kinetic energy should be much smaller during the approaching stage. Indeed, as it is seen from Fig. 3, the dissipated energy calculated with the microscopic transport coefficients is on the average almost twice smaller as compared with macroscopic approach. Consequently, at the touching point the incoming ions will still have a larger momentum  $p$  in the fusion direction, what should increase the fusion probability.

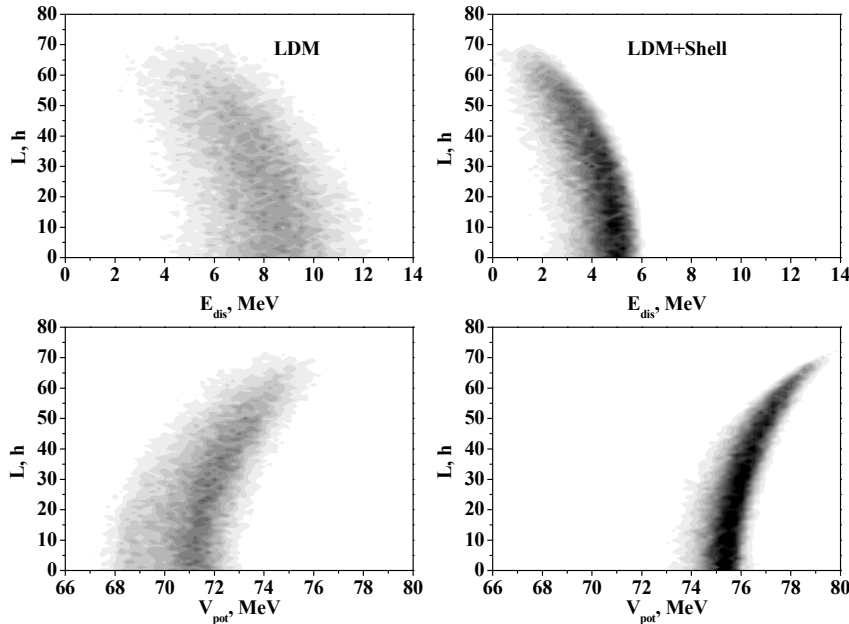


Fig. 3. The distribution of the touching probability  $T_L$  in the angular momentum  $L$  (in units of  $\hbar$ ) and the dissipated (top) and potential (bottom) energy at the touching point for the reaction  $^{18}\text{O} + ^{208}\text{Pb} \rightarrow ^{226}\text{Th}$ .

The comparison of the touching probability distributions calculated with and without shell effects is presented in Fig. 3. As it was mentioned above, due to the larger stiffness of the potential energy the distribution both in the dissipated  $E_{dis}$  and potential  $V_{pot}$  energies got more narrow (left side). Besides, the distribution in  $V_{pot}$  is shifted by approximately 4 MeV to the larger energies.

From the calculations of the touching probability for the reaction  $^{100}\text{Mo} + ^{100}\text{Mo} \rightarrow ^{200}\text{Po}$  one can estimate the effect of different initial orientations of the deformed ions. In case of initially spherical ions only the liquid drop part of the deformation energy was taken into account. It turns out that the most favourable orientation is “nose to nose”, i.e. when both ions are initially prolate (Fig. 4). In this case the touching probability is the largest. The combination when one ion is oblate and the other is prolate initially has somewhat smaller touching probability. The smallest probability is obtained in the case when both ions are

initially oblate. The touching probability of the initially spherical ions is in between of the last two cases.

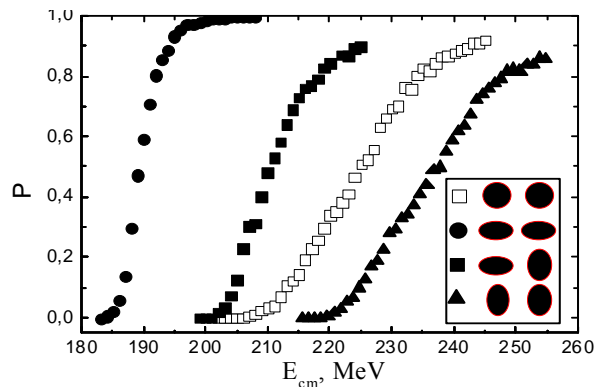


Fig. 4. The dependence of the touching probability  $P$  on the initial energy  $E_{cm}$  for the reaction  $^{100}\text{Mo} + ^{100}\text{Mo} \rightarrow ^{200}\text{Po}$ . The circles, filled and open squares, triangles correspond to initial deformations of the ions: prolate-prolate, oblate-prolate, spherical-spherical and oblate-oblate.

The shift of the touching distributions in the potential energy at the touching point is of the order of 15 MeV. I.e., neglecting the ground state deformation (due to the shell structure) may lead to the error in the potential energy at the touching point of the order of 15 MeV.

### Conclusions

In the present work the formulated earlier [4] the two stage model of fusion-fission reactions is extended by the account of the shell structure of colli-

ding nuclei. The numerical calculations are carried out for the two reactions: with deformed ( $^{100}\text{Mo} + ^{100}\text{Mo} \rightarrow ^{200}\text{Po}$ ) and spherical ( $^{208}\text{Pb} + ^{18}\text{O} \rightarrow ^{226}\text{Th}$ ) in ground state ions. It is shown that the shell effects have quite a noticeable influence on the dependence of the touching probability on the potential and dissipated energy.

The obtained distributions of the touching probability will be used as the initial conditions for the calculation of the time evolution of the compound system.

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## ЗЛИТТЯ ВАЖКИХ ЯДЕР У РАМКАХ ДВОЕТАПНОЇ МОДЕЛІ РЕАКЦІЙ

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Розглянуто процес зближення іонів у реакціях злиття-поділу. Стан сукупної системи характеризується трьома колективними параметрами: параметрами квадрупольної деформації ядра-мішені та налітаючого іона і віддаллю між їхніми центрами мас. Ми припускаємо, що мішень та іон орієнтовані «ніс до носу». Динаміка процесу описується за допомогою рівнянь Ланжевена для наведених вище колективних параметрів. У розрахунках енергії деформації мішені та іона враховується їхня оболонкова структура. Чисельні розрахунки виконані для деформованих ( $^{100}\text{Mo} + ^{100}\text{Mo} \rightarrow ^{200}\text{Po}$ ) та сферичних ( $^{208}\text{Pb} + ^{18}\text{O} \rightarrow ^{226}\text{Th}$ ) в основному стані іонів. Показано, що оболонкові ефекти в енергії іонів впливають не тільки на процес у цілому (на висоту бар'єра злиття, залежність імовірності торкання від початкової кінетичної енергії), але й на стан кожного іона (на їхню форму та енергію збудження).

*Ключові слова:* реакції злиття-поділу, рівняння Ланжевена, енергія деформації, оболонкова структура.

## СЛИЯНИЕ ТЯЖЕЛЫХ ЯДЕР В РАМКАХ ДВУХШАГОВОЙ МОДЕЛИ РЕАКЦИЙ

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Рассмотрен процесс сближения ионов в реакциях слияния-деления. Состояние совокупной системы описывается тремя коллективными параметрами: параметрами квадрупольной деформации ядра-мишени и налетающего иона и расстоянием между их центрами масс. Мы предполагаем, что мишень и ион ориентированы «нос к

носу». Динамика процесса описывается с помощью уравнений Ланжевена для указанных выше коллективных параметров. При расчете энергии деформации мишени и иона учитывается их оболочечная структура. Численные расчеты выполнены для деформированных ( $^{100}\text{Mo} + ^{100}\text{Mo} \rightarrow ^{200}\text{Po}$ ) и сферических ( $^{208}\text{Pb} + ^{18}\text{O} \rightarrow ^{226}\text{Th}$ ) в основном состоянии ионов. Показано, что учет оболочечных эффектов в энергии ионов влияет не только на процесс в целом (на высоту барьера слияния, зависимость вероятности касания от начальной кинетической энергии), но и на состояние каждого иона (на их форму и энергию возбуждения).

*Ключевые слова:* реакции слияния-деления, уравнения Ланжевена, энергия деформации, оболочечная структура.

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