ЯДЕРНА ФІЗИКА

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EFFECTS OF CLUSTER POLARIZATIONS ON THE RADIATIVE CAPTURE REACTIONS $^{3}He(\alpha,\gamma)^{7}Be$, $^{3}He(\alpha,\gamma)^{7}Li$, $^{6}Li(p,\gamma)^{7}Be$ AND $^{6}Li(n,\gamma)^{7}Li$

The microscopic three-cluster model, developed by the authors, was applied to study effects of cluster polarization on the capture reactions $^{3}He(\alpha,\gamma)^{7}Be$, $^{3}H(\alpha,\gamma)^{7}Li$, $^{6}Li(p,\gamma)^{7}Be$ and $^{6}Li(n,\gamma)^{7}Li$. These reactions are of great importance for the astrophysical applications. Thus main attention is devoted to the cross section (or astrophysical S factor) of the reactions at the low-energy range. We also study in detail correlations between astrophysical S factor of the reactions at zero energy and different quantities associated with the ground state of compound nucleus.

Keywords: three-cluster model, cluster polarization, capture reaction, astrophysical S factor.

Introduction

The aim of the paper is to study how strongly cluster polarization effects cross section or astrophysical S factor of the capture reactions $^{3}He(\alpha,\gamma)^{7}Be$, $^{3}H(\alpha,\gamma)^{7}Li$, $^{6}Li(p,\gamma)^{7}Be$ and $^{6}Li(n,\gamma)^{7}Li$ in ^{7}Be and ^{7}Li nuclei. This investigation are stimulated by two factors.

First, the radiative capture and photonuclear reactions are a source of interesting and valuable information about the dynamics and structure of nuclear systems. This information is of great importance for fundamental and applied investigations. It well known that cross section of the radiative capture and photodisintegration reactions (within the standard approximations) are determined by wave functions of bound and continuous spectrum states of a compound nucleus. Thus these reactions are good testing site for numerous microscopic and semi-microscopic models to check quality of wave functions obtained within the models. And this test verifies both the internal and asymptotic parts of wave functions. From other side, these reactions are a basic stage of processes unfolding inside the Sun and other stars and in the Universe. Astrophysical aspects of the reactions under consideration (connected, for instance, with the problem of solar neutrino and abundance of light elements at the Universe after the Big Bang) are thoroughly discussed in [1 - 5]. Thus, theoretical analysis of the reactions are of great importance for understanding and revealing main factors, which have a great impact on the processes, and for prediction behavior of cross section of these reactions to the energy range which dominates in the Sun and Universe. Numerous experiments have been performed [6 - 17] to determine the astrophysical S factor of the reactions at energies which are relevant to astrophysical applications. And huge theoretical efforts have been applied within microscopic and semi-microscopic methods (see, for instance, [18 - 29]) to analyze these reactions and to establish general features or to reveal main factors which are of a great importance.

Second, in our recent papers [30, 31] we formulated a microscopic three-cluster model which was specially designed to take into account polarizability of interacting clusters. We called it as cluster polarization. It was shown that polarization of nuclei ^{3}He (^{3}H) and ^{6}Li, which were considered as two-cluster systems: $^{3}He = d + p$, $^{3}H = d + n$ and $^{6}Li = {}^{4}He + d$, play an important role and effects to the great extend position of bound and resonance states in ^{7}Be and ^{7}Li nuclei. It was demonstrated that the smaller is binding energy (with respect to the lowest two body disintegration threshold) of interacting cluster, the larger is polarizability of the cluster. We demonstrated that cluster polarization increases interaction between clusters and thus results in increasing of the binding energy for bound states and substantial decreasing of energy and width of resonance states.

Based on the arguments, which were put forward above, we decided to study the capture reactions in ^{7}Be and ^{7}Li nuclei. The main attention will be devoted to investigation of polarizability of the clusters involved in these reactions. In our early papers [32 - 34] we demonstrated that the collective monopole and quadrupole polarizations have very large effects on the radiative capture and photodisintegration reactions. In the present paper we consider more important and prominent type of polarization, as it was shown in [30, 31]. As in [30, 31], we will use three-cluster configuration $^4He+d+p$ ($^4He+d+n$) to consider binary channels $^4He+^3He$ and ^6Li+p ($^4He+^3H$ and ^6Li+n) in ^7Be (^7Li). And as in [30, 31] we take into account polarizability of clusters ^6Li and ^3He (^3H) which are represented by the two-cluster configuration ^4He+d and $d+p$ ($d+n$) respectively. We assume that polarizability of these clusters effects cross section or the astrophysical S factor of the capture reactions at low energy range.

The paper is organized in the following way. In first section we briefly consider main ideas of the microscopic model designed to take into account cluster polarization. Some basic formulae are discussed which we use to calculated the capture reactions. In the second section we justify our choice of the input parameters and present main results for bound and continuous spectrum states. Detail analysis of effects of the cluster polarization on the astrophysical S factor of the capture reactions are carried out in this section.

Model formulation

We slightly modify the microscopic model, details of which are presented in [30, 31]. The novelty of the present model is that we included mixture of states with different total spin S and total orbital momentum L. In [30, 31] we restricted ourselves with the total spin $S=1/2$ and thus the total orbital momentum was a good quantum number. In present paper we involve states with values of the total spin $S=1/2$ and $S=3/2$. For instance, the ground state of ^7Be or ^7Li with total angular momentum $J=3/2$ will be presented by the following combination of total spin S and total orbital momentum L: $(L,S)=(1,1/2)+(1,3/2)$. Note that channels with the total spin $S=3/2$ are realized when spins of deuteron and proton (neutron) are aligned at the same direction and thus within model suggested there is no bound state in ^3He (^3H). Or, in other words, this spin state gives no contribution to the bound state of ^3He (^3H). Trail wave function has a form

$$\Psi^J = \hat{A}\left\{\left[\Phi_1(A_1)\Phi_2(A_2)\Phi_3(A_3)\right]_S\left[f_1(\mathbf{x}_1,\mathbf{y}_1)+f_2(\mathbf{x}_2,\mathbf{y}_2)+f_3(\mathbf{x}_3,\mathbf{y}_3)\right]_L\right\}_J, \quad (1)$$

where $\Phi_\alpha(A_\alpha)$ is a shell-model wave function for the internal motion of cluster α ($\alpha=1,2,3$) and $f_\alpha(\mathbf{x}_\alpha,\mathbf{y}_\alpha)$ is a Faddeev amplitude. In Table 1 we define all values connected with the Faddeev amplitudes. Note that the Jacobi vector $\mathbf{x}_\alpha$ connects center of mass of clusters indicated in the column "Two-cluster system", while the Jacobi vector $\mathbf{y}_\alpha$ determines distance between clusters indicated in the column "Binary channel". We wish to underline that the binary channels, mentioned in Table 1, are dominant binary configurations in ^7Be and ^7Li. They are obtained by projection of three-cluster configurations $^4He+d+p$ and $^4He+d+n$ onto space of two-cluster configurations. This projection is a necessary step to incorporate proper boundary conditions which is of paramount importance for scattering states. Clusters, indicated in the column "Two-cluster system", are subject for cluster polarization. They are represented as two-cluster subsystems.

Table 1. **Correspondence between the Faddeev amplitudes and binary channels in ^7Be and ^7Li**

	^7Be		^7Li	
Amplitude	Binary channel	Two-cluster system	Binary channel	Two-cluster system
$f_1(\mathbf{x}_1,\mathbf{y}_1)$	$^4He+^3He$	$^3He=d+p$	$^4He+^3H$	$^3H=d+n$
$f_2(\mathbf{x}_2,\mathbf{y}_2)$	^6Li+p	$^6Li=^4He+d$	^6Li+n	$^6Li=^4He+d$
$f_3(\mathbf{x}_3,\mathbf{y}_3)$	^5Li+d	$^5Li=^4He+p$	^5He+d	$^5He=^4He+n$

One notice, that wave function ((1)) is written for the LS coupling scheme, when total spin S is vector sum of spin individual clusters, total orbital momentum L is vector sum of partial orbital momenta and total angular momentum is $\mathbf{J}=\mathbf{L}+\mathbf{S}$. This scheme can be used to calculate spectrum of three-cluster system. However, one has to use the JJ coupling scheme to study continuous spectrum states.

To proceed further we need to construct a wave function of two-cluster subsystems. We denote this function as $\Psi_\alpha^{J_\alpha}$ and represent as

$$\Psi_\alpha^{J_\alpha} = \hat{A}_\alpha\left\{\left[\Phi_\beta(A_\beta)\Phi_\gamma(A_\gamma)\right]_{S_\alpha}\left[g_\alpha(\mathbf{x}_\alpha)\right]_{\lambda_\alpha}\right\}_{J_\alpha}, \quad (2)$$

where $\hat{A}_\alpha$ is the antisymmetrization operator for two-cluster system. This system consists from two clusters with numbers β and γ. (The indexes α, β and γ form a cyclic permutation of 1, 2 and 3). Function $\Psi_{\alpha}^{J_\alpha}$ has to be determined by solving the Schrödinger equation for particular two-cluster subsystem. By solving this equation, one obtain spectrum and wave functions of bound state(s) if any and pseudo-bound states. These pseudo-bound states, being specific states of two-cluster continuos spectrum, allow one to study flexibility or polarizability of the two-cluster compound system.

In [30, 31] we made use of the Gaussian basis to expand unknown function $g_\alpha(\mathbf{x}_\alpha)$ and to reduce the Schrödinger equation for two-cluster subsystem to a simple matrix form, which can be easily solved numerically. The advantage of the Gaussian basis is that it allows to describe bound states of weakly bound nuclei with minimal set of functions.

Let us numerate eigenstates of two-cluster hamiltonian by index σ. Thus we have got energy $E_\sigma^{(\alpha)}$ and wave functions $g_{\sigma\alpha}(\mathbf{x}_\alpha)$. Having got the set of two-cluster functions $g_{\sigma\alpha}(\mathbf{x}_\alpha)$, we can use them to expand three-cluster function:

$$\Psi^J = \sum_\alpha \sum_\sigma \hat{A}\left\{\left[\Phi_1(A_1)\Phi_2(A_2)\Phi_3(A_3)\right]_S \left[g_{\sigma\alpha}(\mathbf{x}_\alpha)\varphi_{\sigma\alpha}(\mathbf{y}_\alpha)\right]_L\right\}_J, \quad (3)$$

This is typical and rather popular way of solving many-particle problems.

Now we have to determine a set of wave functions $\varphi_{\sigma\alpha}(y_\alpha)$, which connected with relative motion of a cluster with number α with respect to center of mass of two-cluster subsystem, formed by clusters with numbers β and γ. Sum of energy ε_α of internal motion of cluster α (ε_α is eigenenergy of a single cluster hamiltonian) and energy $E_\sigma^{(\alpha)}$ of two-cluster system determine energy of two-body threshold. Thus by using expansion (3), we reduced our three-cluster system to the set of coupled multi-channel two-body systems. In these three-cluster and two-body systems, the Pauli principle is treated in exact manner and interaction within each two-body system and coupling of the channels are determined by superposition of nucleon-nucleon potential.

It should be note that the present model provides more advanced description of internal structure of clusters, which are described by functions (3) and listed in Table 1 in the column "Two-cluster system". If we take only function of bound state(s) of two-cluster subsystem, we have got rigid or nonflexible clusters. They don't change their shape and size while interacting with a third cluster. If one or more wave functions of pseudo-bound states are involved, we have got flexible cluster and thus have got all meanings to study cluster polarization. In what follows we will distinguish two cases. First case corresponds to a rigid cluster and it will be denoted as "N", which means that polarizations is not taken into account for selected two-cluster subsystem. In this case only one function $g_{\sigma\alpha}(\mathbf{x}_\alpha)$ with $\sigma = 0$ is involved in calculations. Second case is denoted by "Y" says that polarizability of the subsystem is taking into account. In this case all functions $g_{\sigma\alpha}(\mathbf{x}_\alpha)$ are involved in calculations.

To determine energy and wave functions of bound states or S-matrix and wave functions of continuous spectrum states, we make use of the oscillator basis. It incorporated to expand functions $\varphi_{\sigma\alpha}(y_\alpha)$ and to simplify solution of the Schrödinger equation by reducing it to the set of linear equations. Main advantage of the oscillator basis is that it allows to impose proper boundary conditions for bound and scattering states (see details in [35 - 38]).

Cross section of the radiative capture for electric transition of the polarity λ is

$$\sigma(E) = \frac{8\pi}{\hbar}\frac{k_\gamma^{2\lambda+1}}{(2S_1+1)(2S_2+1)}\frac{(\lambda+1)}{\lambda\left[(2\lambda+1)!!\right]^2}\frac{(2J_f+1)}{(2l_i+1)}\sum_S\left|\left\langle \Psi_{l_f}^{J_f}\left\|\hat{M}_{E\lambda}\right\|\Psi_{l_i S}^{J_i}(E)\right\rangle\right|^2, \quad (4)$$

where S_1 and S_2 are spins of incident clusters, J_i and J_f stand for total angular momenta of incident and final states respectively, l_i and l_f denote the orbital momentum of relative motion of two clusters in initial and final states. Thus to calculated cross section or the astrophysical S factor, which is convenient to use at astrophysical energies and is related to the cross section by the relation

$$S(E) = \sigma(E)E\exp\{2\pi\eta\}$$

($\eta = Z_1Z_2e^2/\hbar v$ is the Sommerfeld parameter), one need to calculated matrix element of the operator $\hat{M}_{E\lambda}$ between wave functions of initial and final states. The electric λ-pole operator

$$\hat{M}_{E\lambda} = e\sum_{i=1}^{A}\frac{1}{2}(1+\hat{\tau}_{iz})r_i^\lambda Y_{\lambda\mu}(\hat{\mathbf{r}}_i),$$

here $\mathbf{r}_i$ is a coordinate of i th nucleon in the center mass system. It is assumed in Eq. (4) that the wave function of bound state is normalized by the condition

$$\left\langle \Psi_{l_f}^{J_f} \mid \Psi_{l_f}^{J_f} \right\rangle = 1$$

and wave function of continuous spectrum is of the unit flux

$$\Psi_{l_i S_i}^{J_i}(E) = \sqrt{\frac{\pi(2l_i+1)}{v}} \sum_{M_i} C_{l_i 0; S_i M_S}^{J_i M_i} \hat{A} \left\{ \left[\left[\Phi_1(A_1) \Phi_2(A_2) \right]^S Y_{l_i}(\hat{\mathbf{y}}) \right]^{J_i M_i} g_{l_i J_i}(y) \right\} \quad (5)$$

and v is the relative velocity between two incident nuclei.

More details about calculations cross section of the radiative capture or photodisintegration can be found, for instance, in [20, 27, 28, 39].

In this paper we take into account only electrical dipole and quadrupole transitions which, as was shown repeatedly, dominate at the low energy region.

Results

To find spectrum and wave functions of discrete and continuos spectrum states of ^7Be and ^7Li, we employ the Minnesota potential (central part is taken from [40] and spin-orbital one is from [41] (set number IV)). In [30, 31] parameter u of the potential was chosen to reproduce experimental difference between the $^4He+^3He$ and ^6Li+p ($^4He+^3H$ and ^6Li+n) threshold energies. It was done in order to be consistent with the experimental situation for the reactions $^6Li(p,^3He)^4He$ and $^6Li(n,^3H)^4He$. In this paper we use the same value of u. As in [30, 31], we chose the oscillator length, which is common for deuteron and alpha-particle, to minimize energy of the three-cluster threshold $^4He+d+p$ ($^4He+d+n$).

We make use of 4 Gaussian functions and 130 oscillator functions to construct wave functions of bound and continuous spectrum states of ^7Be and ^7Li. In [30, 31] and in this paper we made sure that this amount of basis functions is large enough to provide convergent solutions for scattering states and cross section of the nuclear rearrangement processes and radiative capture reactions. In what follows we present four types of calculations which will be distinguished by two letters: (N, N), (Y, N), (N, Y) and (Y, Y). First letter indicates whether polarization of ^3He (^3H) is regarded (Y) or disregarded (N) in calculations. Second letter is associated with polarization of ^6Li cluster in the same way.

We start our calculations from the ground $J^\pi = 3/2^-$ state of ^7Be and ^7Li, we find out energy and wave function of the state, and than calculate the quadrupole moment, the r.m.s. proton, neutron and matter radii. We also determine a spectroscopic factor (SF) (see definition of the SF, for instance, in [42]) for clusterization $4+3$ and $6+1$. We obtain energy and wave function of the first exited $J^\pi = 1/2^-$ state and evaluate the $B(E2)$ transition probability from this state to the ground state. These quantities and correlation between them will be discussed later at the end of this section. We begin discussion with results of calculations of the S factor for the reactions $^3He(\alpha,\gamma)^7Be$, $^3H(\alpha,\gamma)^7Li$, $^6Li(p,\gamma)^7Be$ and $^6Li(n,\gamma)^7Li$.

In Figs. 1 and 2 we show effects of cluster polarization on the S factor of the reactions $^3He(\alpha,\gamma)^7Be$ and $^3H(\alpha,\gamma)^7Li$ at the energy range $0 \le E \le 1$ MeV in the entrance channel. In these and other figures we display only the dipole transition from the $1/2^+$ continuous spectrum state to the ground state of nucleus.

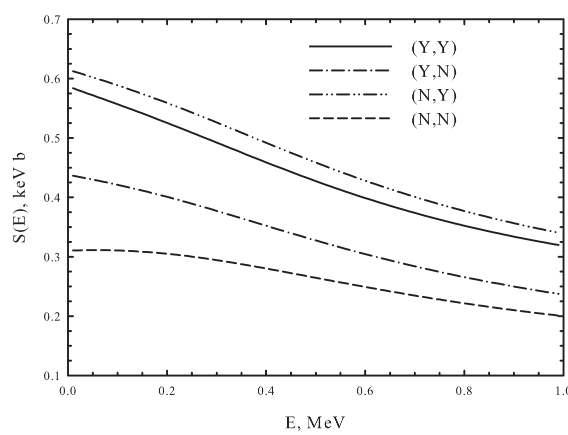

Fig. 1. The astrophysical factor for the reaction $^3He(\alpha,\gamma)^7Be$ with effect of cluster polarization.

One immediately notices that cluster polarization effects to a great extent astrophysical S factor of the reactions $^3He(\alpha,\gamma)^7Be$ and $^3H(\alpha,\gamma)^7Li$. It changes not only the S factor at zero energy but it also change dependence of the S factor on energy at low energy range. For both reactions, effects of polarization of ^6Li is more stronger than polarization of ^3He or ^3H.

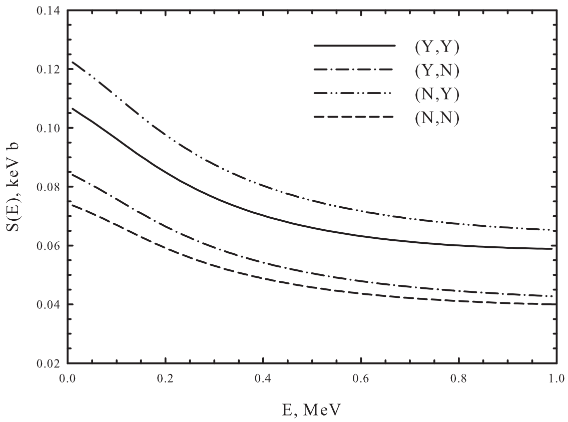

Fig. 2. Effects of cluster polarization on S factor of the reaction ${}^3H(\alpha,\gamma)^7Li$.

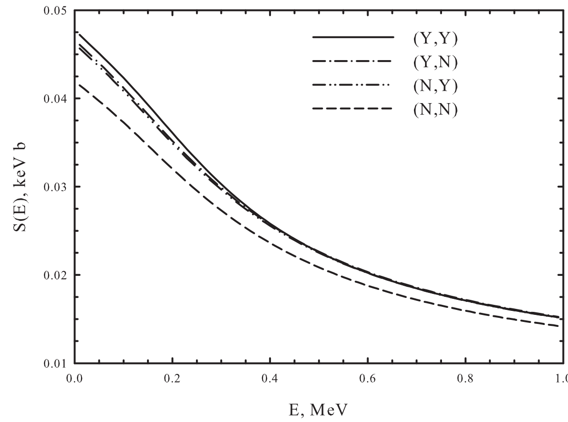

Fig. 3. Effects of cluster polarization on the reaction ${}^6Li(p,\gamma)^7Be$.

Effects of cluster polarization on astrophysical S factor of the reaction ${}^6Li(p,\gamma)^7Be$ are shown in Fig. 3.

One can see that in this case influence of the cluster polarization is not so strong as for the reactions ${}^3He(\alpha,\gamma)^7Be$ and ${}^3H(\alpha,\gamma)^7Li$. Besides, effects of cluster polarization on the reaction ${}^6Li(n,\gamma)^7Li$ is much more smaller than for the reaction ${}^6Li(p,\gamma)^7Be$. We do not display figure for this reaction. Note, that a similar picture was observed in [30, 31] for the reactions ${}^6Li(p,\alpha)^3He$ and ${}^6Li(n,\alpha)^3H$: cluster polarization effected more cross section of the reaction ${}^6Li(p,\alpha)^3He$ than cross section of the reaction ${}^6Li(n,\alpha)^3H$.

Table 2. **Correlations between astrophysical S factor $S(0)$ and energy of the ${}^7Be$ ground state $E\left(\frac{3^-}{2}\right)$, quadrupole moment Q, the r.m.s. proton radius R_p, spectroscopic factor SF for the clusterization ${}^4He+{}^3He$ and ${}^6Li+p$**

Polarization		${}^7Be$					
${}^3He$	${}^6Li$	$S(0)$, keV b	$E\left(\frac{3^-}{2}\right)$, MeV	Q, e fm^2	R_p, fm	$SF(4+3)$	$SF(6+1)$
N	N	0.310	-1.234	-7.286	2.570	1.008	0.781
Y	N	0.438	-1.808	-6.507	2.461	0.993	0.747
N	Y	0.615	-2.173	-6.469	2.381	0.983	0.734
Y	Y	0.587	-2.212	-6.513	2.396	0.981	0.733

Table 3. **Correlations between the zero-energy S factor of the reaction ${}^4He({}^3H,\gamma)^7Li$ and energy of the ${}^7Li$ ground state $E\left(\frac{3^-}{2}\right)$, the quadrupole moment Q, the r.m.s. proton radius R_p, spectroscopic factor SF for the clusterization ${}^4He+{}^3H$ and ${}^6Li+n$**

Polarization		${}^7Li$					
${}^3H$	${}^6Li$	$S(0)$, keV b	$E\left(\frac{3^-}{2}\right)$, MeV	Q, e fm^2	R_p, fm	$SF(4+3)$	$SF(6+1)$
N	N	0.075	-2.047	-4.045	2.349	1.018	0.784
Y	N	0.085	-2.653	-3.609	2.248	1.002	0.746
N	Y	0.124	-3.033	-3.711	2.173	0.991	0.732
Y	Y	0.108	-3.075	-3.756	2.179	0.990	0.731

Tables 2 and 3 and Figs. 1, 2 and 3 reveal nonlinear effects of cluster polarization on the S factor of the capture reactions under investigations. Polarization of ^{6}Li and ^{3}He, taking into account separately (case - (Y, N), and case - (N, Y)), increases the S factor, obtained without polarization (case - (N, N)). And having this results, one may expect that the more polarization, the large is the S factor of the capture reactions. However, if we involve both polarization simultaneously (case - (Y, Y)), then the S factor is decreases (one need to compare it with the case - (N, Y)).

This conclusion is also confirmed by considering dependence of the zero-energy S factor on parameters of the ground state. In Figs. 4, 5, 6 and 7 we demonstrate correlations between the zero-energy S factor ($S(0)$) and energy $(E_{g.s.})$, the r.m.s. proton radius (R_p) and the quadrupole moment (Q) of the ground state. If we take three cases, namely (N, N), (Y, N) and (Y, Y), we observe simple, almost linear correlations between $S(0)$ factor and energy $E_{g.s.}$, the radius R_p, the quadrupole moment Q of the ground state. Polarization of ^{6}Li, taking into consideration alone, violates such a simple correlation.

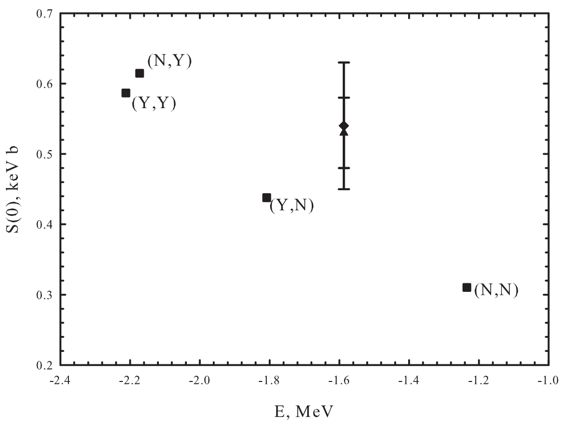

Fig. 4. Correlation between astrophysical S factor of the reaction $^{4}He\left(^{3}He,\gamma\right)^{7}Be$ and energy of ^{7}Be ground state. Error bar marks correlation between experimental values of S factor and energy of the ground state.

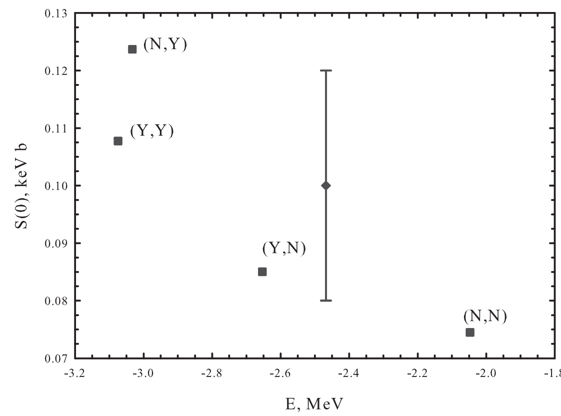

Fig. 5. Correlation between astrophysical S factor of the reaction $^{4}He\left(^{3}H,\gamma\right)^{7}Li$ and energy of ^{7}Li ground state.

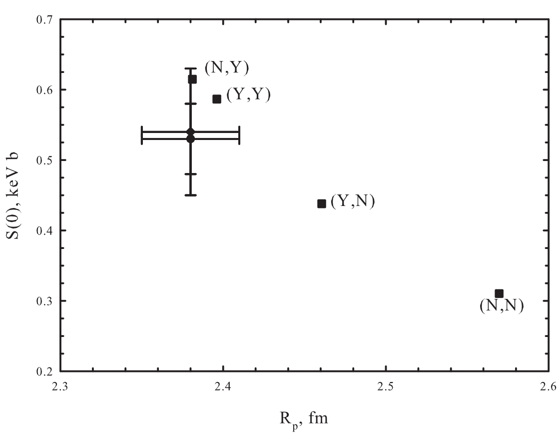

Fig. 6. The zero-energy S factor of the reaction $^{4}He\left(^{3}He,\gamma\right)^{7}Be$ as a function of the r.m.s. proton radius.

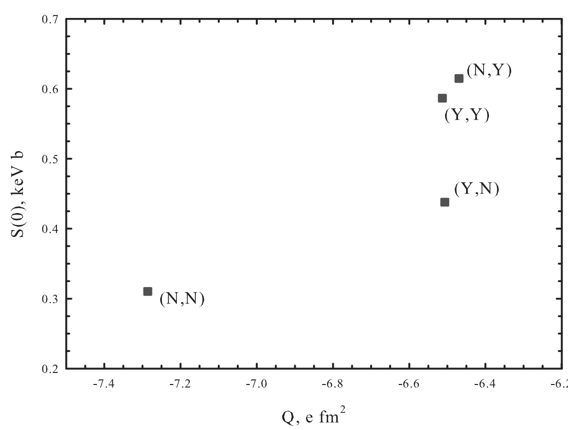

Fig. 7. Astrophysical S factor of the reaction $^{4}He\left(^{3}He,\gamma\right)^{7}Be$ as a function of the quadrupole moment of the ground state.

In Figs. 4, 5 and 6 we display not only theoretical values of $S(0)$, $E_{g.s.}$, R_p, but also available experimental data. The ground state energy of ^{7}Be and ^{7}Li nuclei is taken from [43]. The proton radius of ^{7}Be was determined in [44]. Analysis of the experimental data, carried out by Angulo et al in [1] and by Adelberger et al in [2], yielded two adopted values for the S factor of the reaction $^{3}He(\alpha,\gamma)^{7}Be$ which are equal $S(0) = 0.54 \pm 0.09$ keV b and $S(0) = 0.53 \pm 0.05$ keV b respectively. We use both of these values for $S(0)$. The recommended value of the zero-energy S factor for the reaction $^{3}H(\alpha,\gamma)^{7}Li$ is $S(0) = 0.10 \pm 0.02$ keV b and was deduced in [1].

Note that some types of these correlations have been discussed in literature. For instance, Kajino [25] investigated correlations between the S factor for the $^{3}He(\alpha,\gamma)^{7}Be$ reaction and r.m.s. proton radius and the quadrupole moment of ^{7}Be, calculated within two-cluster microscopic model with 7 different nucleon-nucleon potentials. He demonstrated approximate linear correlations between these quantities. In [18] Csótó and Langanke used the two-cluster extended model to calculate the S factor of the reactions $^{3}He(\alpha,\gamma)^{7}Be$ and $^{3}H(\alpha,\gamma)^{7}Li$. They considered correlation between the zero-energy S factor and the quadrupole moment Q. Different values of $S(0)$ and Q were obtained by varying parameter u of the Minnesota potential, the Majorana parameter m of the modified Hasegawa - Nagata potential and size parameter of interacting clusters.

We have no room to discuss other correlations for reactions $^{3}He(\alpha,\gamma)^{7}Be$ and $^{3}H(\alpha,\gamma)^{7}Li$ and to say nothing of the reactions $^{6}Li(p,\gamma)^{7}Be$ and $^{6}Li(n,\gamma)^{7}Li$. That is why in Table 2 we summarize results for ^{7}Be, obtained within the model, to demonstrate correlations between value of the S factor for the reaction $^{3}He(\alpha,\gamma)^{7}Be$ at zero energy and other quantities, connected with the ground state. The same quantities for the ^{7}Li ground state and for the S factor of the reaction $^{3}H(\alpha,\gamma)^{7}Li$ are displayed in Table. 3

One can deduce visually or qualitatively from Figs. 1, 2 and 3 that polarization effects very much the astrophysical S factor of the radiative capture reactions. Tables 2 and 3 allow us to quantify these effects. Note that, like in [30, 31], polarization of ^{6}Li is more pronounced than polarization of ^{3}He (^{3}H). Indeed, if we compare S factor at zero energy for the reaction $^{3}He(\alpha,\gamma)^{7}Be$ with polarization (case - (Y, N), and case - (N, Y)) and without polarization (case - (N, N)), we see that polarization of ^{6}Li almost double $S(0)$, while polarization of ^{3}He increases the zero-energy S factor 1.41 times. For the reaction $^{3}H(\alpha,\gamma)^{7}Li$, polarization of ^{6}Li increases $S(0)$, obtained without polarization, 1.65 times, and polarization of ^{3}H increases it only 1.13 times.

Conclusion

We have advanced the three-cluster microscopic model formulated in [30, 31] to study the radiative capture reactions in ^{7}Be and ^{7}Li nuclei. We extended Hilbert space and included states with different values of the total spin and orbital momentum. The selected three-cluster configurations allowed to take into account the dominant binary channels ^{7}Be and ^{7}Li nuclei. Moreover, the configurations also allowed to consider cluster structure of interacting clusters and thus provide a realistic description these clusters.

Effects of cluster polarization have been investigated in detail. It was shown that cluster polarization effects very much cross section of the capture reactions $^{3}He(\alpha,\gamma)^{7}Be$, $^{3}H(\alpha,\gamma)^{7}Li$, $^{6}Li(p,\gamma)^{7}Be$ and $^{6}Li(n,\gamma)^{7}Li$. Polarization of ^{6}Li cluster had a more stronger impact on the cross section of the radiative capture reactions, than polarization of ^{3}He (^{3}H) cluster. And this is true for the reaction $^{6}Li(p,\gamma)^{7}Be$, when $^{6}Li+p$ channel is open and dominant, and also for the reaction $^{3}He(\alpha,\gamma)^{7}Be$ where this channel is closed. We have discovered that the reactions $^{3}He(\alpha,\gamma)^{7}Be$ and $^{3}H(\alpha,\gamma)^{7}Li$ were much more strongly effected by cluster polarization, than the reactions $^{6}Li(p,\gamma)^{7}Be$ and $^{6}Li(n,\gamma)^{7}Li$.

We have investigated correlations between the astrophysical S factor of the reactions $^{3}He(\alpha,\gamma)^{7}Be$ and $^{3}H(\alpha,\gamma)^{7}Li$ at zero energy $S(0)$ and the r.m.s. proton radius and the quadrupole moment of the bound state, the spectroscopic factor for clusterization $^{4}He+^{3}He$ (^{3}H) and $^{6}Li+p$ ($^{6}Li+n$). There was observed almost linear dependence of the zero-energy S factor on these quantities in our calculations.

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ЕФЕКТИ КЛАСТЕРНОЇ ПОЛЯРИЗАЦІЇ НА РЕАКЦІЇ РАДІАЦІЙНОГО ЗАХОПЛЕНННЯ $^3He(\alpha,\gamma)^7Be$, $^3H(\alpha,\gamma)^7Li$, $^6Li(p,\gamma)^7Be$ та $^6Li(n,\gamma)^7Li$

Мікроскопічна трикластерна модель, яка була запропонована авторами раніше, залучається для дослідження впливу кластерної поляризації на перерізи реакцій радіаційного захоплення $^3He(\alpha,\gamma)^7Be$, $^3H(\alpha,\gamma)^7Li$, $^6Li(p,\gamma)^7Be$ та $^6Li(n,\gamma)^7Li$. Ці реакції мають важливе значенння для астрофізичних застосувань. Тому головну увагу приділено поведінці перерізів реакцій (або астрофізичних S факторів) в області низьких енергій. Крім цього, детально досліджено кореляцію між S фактором при нульовій енергії та різними величинами, що характеризують основний стан компаунд-ядра.

Ключові слова: трикластерна модель, поляризація кластера, реакція захоплення, астрофізичний S фактор.

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ЭФФЕКТЫ КЛАСТЕРНОЙ ПОЛЯРИЗАЦИИ НА РЕАКЦИИ РАДИАЦИОННОГО ЗАХВАТА $^3He(\alpha,\gamma)^7Be$, $^3H(\alpha,\gamma)^7Li$, $^6Li(p,\gamma)^7Be$ и $^6Li(n,\gamma)^7Li$

Микроскопическая трехкластерная модель, предложенная авторами ранее, привлекается для исследования влияния кластерной поляризации на сечения реакций радиационного захвата $^3He(\alpha,\gamma)^7Be$, $^3H(\alpha,\gamma)^7Li$, $^6Li(p,\gamma)^7Be$ и $^6Li(n,\gamma)^7Li$. Эти реакции имеют важное значение для астрофизических приложений. Поэтому главное внимание уделено поведению сечений реакций (или астрофизическим S факторам) в области низких энергий. Кроме этого, детально исследована корреляция между S фактором при нулевой энергии и различными величинами, характеризующими основное состояние компаунд-ядра.

Ключевые слова: трехкластерная модель, поляризация кластера, реакция захвата, астрофизический S фактор.

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