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NUCLEAR ASYMMETRY ENERGY, NEUTRON SKIN AND ISOVECTOR STIFFNESS

The isovector particle densities and surface tension coefficients for the average binding energy in the approximation of a sharp edge proton-neutron asymmetric nucleus are used for analytical calculations of its neutron skin and isovector stiffness coefficients. They are significantly different from the well-known ones for the most Skyrme forces. The energies and energy-weighted sum rules of the isovector giant dipole resonances obtained within the Fermi-liquid drop model are in good agreement with the experimental data.

Keywords: nuclear binding energy, symmetry surface energy, proton-neutron asymmetry, neutron skin, isovector stiffness, dipole giant resonances.

Introduction

The neutron skin of the exotic nuclei with a large excess of neutrons against protons is still one of the remarkable subjects of the nuclear and astronomic physics [1 - 6]. The simple and accurate solution for the isovector particle density distributions were obtained within the nuclear effective surface (ES) approximation [7 - 11]. It exploits the property of saturation of the nuclear matter and a narrow diffuse-edge region in finite heavy nuclei. The ES is defined as the location of points of the maximum density gradient. The coordinate system related locally to the ES is specified by a distance ξ from the given point to the surface and tangent coordinate η at the ES. The variational condition of the nuclear energy minimum at some fixed integrals of motion in the local energy-density theory is simplified in the ξ, η coordinates. In particular, in the extended Thomas - Fermi (ETF) approach [12, 13] (with the Skyrme forces [14]) it can be done for any deformations by using expansion in a small parameter $a/R \sim A^{-1/3} \ll 1$ for heavy enough nuclei (a is of the order of the diffuse edge thickness of the nucleus, R is its mean curvature radius, and A the number of nucleons). The accuracy of the ES approximation in the ETF approach without spin-orbit (SO) and asymmetry terms was checked [9] by comparing results of the Hartree - Fock (HF) and ETF theories [12] for some Skyrme forces. The ES approach [7 - 9] was extended by accounting for the SO and asymmetry effects [10]. Solutions for the isoscalar and isovector particle densities and energies in the ES approximation of the ETF approach were applied to analytical calculations of the neutron skin and isovector stiffness coefficients in the leading order of the parameter a/R [11]. Our results are compared with the fundamental researches [1 - 3] in the liquid droplet model (LDM). In the present work, we used the derived energy surface constants for calculations of the isovector giant dipole resonances (IVGDR) within the Fermi liquid-drop model (FLDM) [15 - 17].

Asymmetry energy and stiffness

We start with the nuclear energy, $E = \int d\mathbf{r}\, \mathcal{E}(\rho_+(\mathbf{r}), \rho_-(\mathbf{r}))$, in the local density approach [7 - 14],

$$\mathcal{E}(\rho_+, \rho_-) \approx -b_V \rho_+ + J I^2 \rho_+ + \rho_+ \left[\varepsilon_+(\rho_+) - \varepsilon_-\right] +$$

$$+ \left(\mathcal{C}_+ + \mathcal{D}_+ \rho_+\right)\left(\nabla \rho_+\right)^2 + \left(\mathcal{C}_- + \mathcal{D}_- \rho_+\right)\left(\nabla \rho_-\right)^2, \quad (1)$$

where $\rho_\pm = \rho_n \pm \rho_p$ are the isoscalar ρ_+ and isovector ρ_- particle densities, $I = (N - Z)/A$ is the asymmetry parameter, $N = \int d\mathbf{r} \rho_n(\mathbf{r})$ and $Z = \int d\mathbf{r} \rho_p(\mathbf{r})$ are the neutron and proton numbers and $A = N + Z$. As usually, $\mathcal{E}$ (see Eq. (1)) contains the volume and surface terms (two first and three last with the gradient-density ones) [7 - 11], $b_V \approx 16$ MeV is the separation energy per particle and $J \approx 30$ MeV is the volume symmetry-energy constant of the nuclear matter. Eq. (1) can be applied approximately for the most of realistic Skyrme forces [14] by neglecting the relatively small semiclassical $\hbar$ corrections of the ETF kinetic energy and Coulomb terms as shown in [9, 10]. They all can be easily taken into account (without small exchange Coulomb terms) [9, 10]. $\mathcal{C}_\pm$ and $\mathcal{D}_\pm$ are constants defined by the Skyrme force parameters ($\mathcal{D}_-$ is relatively small). The isoscalar surface energy-density part, independent explicitly of the density gradient terms, is determined by the function $\varepsilon_+(\rho_+)$ which satisfies the saturation condition: $\varepsilon_+(\bar{\rho})=0$, $d\varepsilon_+(\bar{\rho})/d\rho_+=0$, where $\bar{\rho}=3/(4\pi r_0^3)\approx 0.16\,\mathrm{fm}^{-3}$ is the density of the infinite nuclear matter, $r_0=R/A^{1/3}$ is a radius constant independent of A. The isovector component can be simply evaluated as $\varepsilon_-=J\left(I^2-\rho_-^2/\rho_+^2\right)$. The isoscalar SO gradient terms in Eq. (1) are defined with a constant: $\mathcal{D}_+=-9mW_0^2/(16\hbar^2)$, where $W_0\approx 100-130\,\mathrm{MeV}\cdot\mathrm{fm}^5$ and m is the nucleon mass (see [12, 14]). From the condition of the minimum energy E under the certain constraints, like the fixed $A=\int d\mathbf{r}\,\rho_+(\mathbf{r})$ and $N-Z=\int d\mathbf{r}\,\rho_-(\mathbf{r})$ one arrives at the Lagrange equations with the isoscalar and isovector multipliers (chemical potentials). To satisfy the condition of the particle number conservation with the required accuracy we account for relatively small surface corrections ($\propto a/R\sim A^{-1/3}$ at the first order) to the leading terms in the chemical potentials [9, 10].

Using the analytical solutions of the Lagrange equations for the isoscalar and isovector particle densities $\rho_\pm$ one obtains [9 - 11] the nuclear energy, $E=E_V+E_S$, in the ES approximation in terms of the volume, $E_V=-b_VA+JI^2A$, and the surface, $E_S=E_S^{(+)}+E_S^{(-)}$, components where

$$E_S^{(\pm)}=\sigma_\pm\mathcal{S}=b_S^{(\pm)}\mathcal{S}/(4\pi r_0^2),$$

$$b_S^{(\pm)}\approx 8\pi r_0^2\mathcal{C}_\pm\int_{-\infty}^{\infty}d\xi\left(1+\mathcal{D}_\pm\rho_+/\mathcal{C}_\pm\right)\left(\partial\rho_\pm/\partial\xi\right)^2, \quad (2)$$

$\mathcal{S}$ is the area of the ES. For the isovector surface energy constant $b_S^{(-)}$ one obtains

$$b_S^{(-)}=k_SI^2, \quad k_S=6\bar{\rho}\mathcal{C}_-\mathcal{J}_-/\left(r_0a\right),$$

$$\mathcal{J}_-=\frac{1}{1+\beta}\int_0^1 dw\,\sqrt{\frac{w\left(1+\beta w\right)}{\varepsilon(w)}}\left[\left(1-w\right)\left(1+\tilde{c}\tilde{w}\right)\right]^2. \quad (3)$$

Here, $a=\sqrt{\mathcal{C}_+\bar{\rho}K/\left(30b_V^2\right)}\approx 0.5\,\mathrm{fm}$ is the diffuseness parameter, $K\approx 230\,\mathrm{MeV}$ is the incompressibility modulus, $\beta=\mathcal{D}_+\bar{\rho}/\mathcal{C}_+$ is the dimensionless SO parameter, $\tilde{w}=\left(1-w\right)/c_{sym}$, $c_{sym}=a\sqrt{J/\left(\bar{\rho}\,|\mathcal{C}_-|\right)}$, $\tilde{c}=\left(\beta c_{sym}/2-1\right)/(1+\beta)$. With the quadratic approximation $\varepsilon(w)\approx(1-w)^2$ one obtains simple expressions for these constants $b_S^{(\pm)}$ (or k_S, see Eqs. (2) and (3)) in terms of the elementary functions.

According to the theory [1 - 3], one can define the isovector stiffness Q with respect to the neutron skin variable τ (the dimensionless measure of the difference between the neutron and proton radii R_n-R_p):

$$E_S^{(-)}=-\frac{\bar{\rho}r_0}{3}\oint d\mathcal{S}\,Q\tau^2\approx-\frac{Q\tau^2\mathcal{S}}{4\pi r_0^2},$$

$$\tau=(R_n-R_p)/r_0. \quad (4)$$

Using also Eq. (2) for the isovector surface energy $E_S^{(-)}$ one may express Q through the isovector surface energy constant k_S as $Q=-b_S^{(-)}/\tau^2=$ $=-k_SI^2/\tau^2$. Defining the neutron and proton ES radii $R_{n,p}$ as the positions of the maxima of the neutron and proton density gradients and expanding in powers of small $R_{n,p}-R$ near the ES up to the first order terms one obtains [11]

$$\tau=\frac{8ag\left(w_r\right)}{r_0c_{sym}^2}I,$$

$$g\left(w\right)=\frac{w^{3/2}\left(1+\beta w\right)^{5/2}}{(1+\beta)\left(3w+1+4\beta w\right)}\times$$

$$\times\left\{w(1+2\tilde{c}\tilde{w})^2+2\tilde{w}(1+\tilde{c}\tilde{w})[\tilde{c}w-c_{sym}(1+2\tilde{c}\tilde{w})]\right\}, \quad (5)$$

where $w_r=w(0)$ is the value of w of the ES, determined by the equation: $\varepsilon(w_r)+w_r(1+$ $+\beta w_r)\varepsilon'(w_r)=0$. Within a good approximation $\varepsilon(w)=(1-w)^2$ [9, 10], one simply has $w_r=(\sqrt{9+8\beta}-3)/(4\beta)$. In Eq. (5), we used also the expressions for the isovector density ρ_- [9, 10]. The neutron and proton particle-density variations conserve the position of the center of mass in the linear approximation in $\delta R_{n,p}$ and asymmetry parameter I. Using Eqs. (3) - (5) one finally arrives at

$$Q=k_S/\tau^2=-\nu J^2/k_S,$$

$$\nu=k_S^2I^2/(\tau^2J^2)=9\mathcal{J}_-^2/\left[16g^2(w_r)\right], \quad (6)$$

where $\mathcal{J}_-$ and $g(w_r)$ are given by Eqs. (3) and (5). Note that the first relationship in Eq. (6) between the isovector quantities, the stiffness Q and the volume J and surface k_S energy constants has the same analytical form as predicted in [1 - 3], $Q=-9J^2/(4k_S)$, where $\nu=9/4$. Its difference from Eq. (6) in terms of J and k_S is in the constant ν which is however proportional to the function $\mathcal{J}_-^2/g^2(w_r)$ in our derivations, instead of 9/4. This function depends significantly on the SO interaction β parameter but not much on the surface asymmetry constant $\mathcal{C}_-$. The constant ν (see Eq. (6)) is weakly sensitive to the specific Skyrme interaction because the most sensitive parameter $\mathcal{C}_-$ was mainly excluded in ν, $\tau \propto 1/c_{sym}^2 \propto \mathcal{C}_-$ and $k_S \propto \mathcal{C}_-$ (see Eqs. (3), (5) and (6)). This constant ν at $\beta = 0$ ($w_r = 1/3$) can be easily evaluated using Eqs. (3), (5) and (6) ($\nu \approx (108/25)\times$ $\times[1-8/(7c_{sym})]^2[1-4/(3c_{sym})]^{-1}$ up to small terms $\propto 1/c_{sym}^2$, $c_{sym} \approx 2-6$ for the Skyrme parameters of [14]). Another difference is the expression (3) itself for k_S. Thus, the isovector stiffness coefficient Q introduced originally by Myers and Swiatecki [1] is not a parameter of our theory but it was found analytically in the explicit closed form Eqs. (6), (3) through the parameters of Skyrme forces.

FLDM and IVGDR

For calculations of the IVGDR we may use the FLDM based on the linearized Landau - Vlasov equations for the dynamical part of distribution functions $\delta f_\pm(\mathbf{r},\mathbf{p},t)$ in the phase space [17],

$$\frac{\partial}{\partial t}\delta f_\pm(\mathbf{r},\mathbf{p},t)+$$

$$+\frac{\mathbf{p}}{m_\pm^*}\nabla_\pm\left[\delta f_\pm(\mathbf{r},\mathbf{p},t)+\delta(e-e_F)\delta e_\pm + V_{ext}^\pm\right]=\delta St_\pm, \quad (7)$$

where $m_\pm^*$ are the isoscalar ($+$) and isovector ($-$) effective masses, $e = p^2/2m_\pm^*$, $e_F = (p_F^\pm)^2/2m_\pm^*$ is the Fermi energy; $p_F^\pm = p_F(1\mp\Delta)$ is the Fermi momenta; $\Delta = 2(1+F_0')I/3$, $F_0' = 3J/e_F - 1$ is the isotropic isovector Landau constant of the quasiparticle interaction; $\delta e_\pm$ is the quasiparticle interaction energies; $V_{ext}^\pm \propto \exp(-i\omega t)$ is the periodic time-dependent external field and $\delta St_\pm \approx -\delta f_\pm/\mathcal{T}$ is the collision term in the simplest $\mathcal{T}$-relaxation time approximation. Solutions of these Eqs. (7) related to the dynamic dipole particle-density variations, $\delta\rho_\pm(\mathbf{r},t) \propto Y_{10}(\hat{r}) \propto \cos(\theta)$ in the spherical coordinates r, θ, φ can be found in terms of a superposition of the plane waves over angles of the wave vector $\mathbf{q}$ as

$$\delta f_\pm = \delta(e-e_F)\times$$

$$\times\int \sin\theta_{\mathbf{q}} d\theta_{\mathbf{q}} d\varphi_{\mathbf{q}} \Phi_\pm Y_{10}(\hat{q}) \exp\left[i(\mathbf{qr}-\omega t)\right]$$

$$\text{with} \quad \hat{q} = \mathbf{q}/q, \quad (8)$$

$\omega = p_F^{(\pm)} s^\pm q/m_\pm^*$, $s^+ = s$, $s^- = s\sqrt{NZ/A^2}$, $q = |\mathbf{q}|$ (the factor $\sqrt{NZ/A^2}$ ensures the conservation of the center-of-mass position, see [18]). $\Phi_\pm$ are the amplitudes of the Fermi surface distortions determined from Eq. (7). The dynamical variations of the quasiparticle interaction $\delta e_\pm(\mathbf{r},\mathbf{p},t)$ at the first order with respect to the equilibrium energy $p^2/2m_\pm^*$ are defined through the particle and current density variations and Landau interaction constants (the isoscalar (F_0) and isovector (F_0') isotropic interaction constants related to the volume incompressibility modulus K and symmetry energy constant J as well as the anisotropic interaction constants corresponding to the effective masses $m_\pm^*$). The two dispersion relations (26) in [17] determine the solutions for the two sounds $s = s_n$ ($n = 1, 2$) as functions of $\omega\mathcal{T}$, the main ($n = 1$) peak and its satellite ($n = 2$) in the nuclear volume due to the nuclear asymmetry.

For the finite Fermi liquid-drop with a sharp ES we may use the macroscopic boundary conditions for the pressures and those for the velocities [10, 11, 17]. For small isovector vibrations near the spherical shape the mean normal velocity u_ξ and normal momentum flux-tensor $\delta\Pi_{\xi\xi}$ components (moments of the distribution function δf_-, see Eq. (8)) are reduced to the radial ones, u_r and $\delta\Pi_{rr}$, respectively,

$$u_r|_{r=R} = u_S.$$

$$\delta\Pi_{rr}|_{r=R} = \delta P_S \quad \text{with}$$

$$\delta P_S = 2\alpha_S^{(-)} b_S^{(-)} \overline{\rho} A^{1/3} Y_{10}(\hat{r})/3. \quad (9)$$

The right hand sides of the boundary conditions are the isovector ES velocity $u_S = R\dot{\alpha}_S^{(-)} Y_{10}(\hat{r})$ and capillary pressure excess δP_S. In Eq. (9), δP_S is given through the isovector surface energy constant $b_S^{(-)} = 4\pi r_0^2 \sigma_-$ [Eq. (3)], $\alpha_S^{(-)}$ is the dynamical isovector-dipole amplitude of the motion of the neutron drop surface against the proton one ($R(t) = R[1+\alpha_S^{(-)}(t)\sqrt{4\pi/5}\,Y_{10}(\hat{r})]$ keeping also the volume and the position of the center of mass conserved).

The energy constant $D = \hbar\omega A^{1/3}$ and energy weighted sum rules (EWSR, $S = -\hbar^2 \int d\omega\omega \,\mathrm{Im}\,\chi(\omega)/\pi$) for the IVGDR can be found from the response function $\chi(\omega)$. Solving the Landau - Vlasov equations (7) in terms of the zero sound plane waves (8) with using the dispersion relations (26) in [17] for s_n and macroscopic boundary conditions (9) on the nuclear ES one obtains

$$\chi(\omega) = \sum_{n=1}^{2} \frac{\mathcal{A}_n(q)}{\mathcal{R}_n(\omega - i\Gamma/2)}, \quad \mathcal{R}_n(\omega) = j_1'(qR) + \frac{3e_F qR}{2k_S A^{1/3}}\left[c_n j_1''(qR) + d_n j_1(qR)\right]. \quad (10)$$

Here $c_1 \approx 1 - 3s_1^2 + F_0'$, $d_1 \approx 1 - s_1^2 + F_0'$ for the main $(n=1)$ IVGDR peak, and more bulky expressions for s_2 of the satellite ($n=2$) peak of a smaller ($\propto I$) strength (see Eq. (D11) in [17]). $\mathcal{A}_1(q) \approx -\bar{\rho}R^3 j_1(qR)/(m\omega^2)$ and $\mathcal{A}_2(q) \propto \Delta$ (Eq. (60) in [17]) are the amplitudes for the $n = 1, 2$ modes, $j_1(z)$ is the standard spherical Bessel function and $j_1'(z) = dj_1/dz$. The poles of the response function $\chi(\omega)$ Eq. (10) (roots ω_n of the equation $\mathcal{R}_n(\omega - i\Gamma/2) = 0$ or q_n) determine the IVGDR energies $\hbar\omega_n$ as their real part (the IVGDR width Γ is determined by their imaginary part). The residue $\mathcal{A}_n$ is important for the calculations of the IVGDR strength (EWSR) by taking the integral of $\omega \mathrm{Im}\,\chi(\omega)$ (see Eq. (10)) at a small width of the IVGDR Γ. Note that the expression like Eq. (10) for the only one main peak in symmetrical nuclei ($N = Z$) with using phenomenological boundary conditions was obtained earlier in [15]. However, in our derivations of Eq. (10), k_S is related to the surface tension coefficient, $\sigma_- = b_S^{(-)}/(4\pi r_0^2) = k_S I^2/(4\pi r_0^2)$, through the isovector capillary pressure δP_S of Eq. (9) and surface energy $E_S^{(-)}$ (Eq. (2)). Therefore, k_S (with the opposite sign) differs essentially from the isovector stiffness coefficient Q (defined through Eq. (4) in [1 - 3]) by τ^2 (see Eq. (6)), in contrast to another interpretation of the corresponding quantity (denoted by B^-) in Eqs. (3) and (20) of [15].

Discussion and summary

The isovector surface energy constants k_S (Eq. (3)), the neutron skin τ (Eq. (5)) and the stiffness coefficients Q (Eq. (6)) in the ES approach using the simplest quadratic approximation for $\varepsilon(w)$ are shown in the Table for several Skyrme forces [14]. The constants k_S (see Eq. (3)) are rather sensitive to the choice of the Skyrme forces. The modulus of k_S for the Lyon Skyrme forces SLy4-7 and SLyb230 [14] is significantly larger than for other forces. Relatively, the stiffness Q is even more sensitive to constants of the Skyrme forces, especially for SGII, than the well-known empiric values $Q \approx 14 - 35$ MeV suggested in [1 - 3]. For T6 [14] one has $\mathcal{C}_- = 0$ and therefore, $k_S = 0$ and $Q = \infty$ (ν is weakly dependent of $\mathcal{C}_-$), in contrast to all other forces shown in the Table. Notice that the isovector gradient terms which are important for the consistent derivations within the ES approach [11] are not also included ($\mathcal{C}_- = 0$) into the energy density in [4, 5]. For RATP [14] the stiffness Q is even negative as $\mathcal{C}_- > 0$ ($k_S > 0$). The reason of significant differences in the Q values might be related to those of the critical isovector Skyrme parameter $\mathcal{C}_-$ in the gradient terms of the energy density (see Eq. (1)). Different experiments used for fitting this parameter were found to be almost insensitive in determining uniquely its value, and hence, k_S (or $b_S^{(-)}$, see Eq. (3)) and Q (see Eq. (6)), as compared to the well-known isoscalar $b_S^{(+)}$ surface-energy constant. The isovector surface-energy constant k_S (see Eq. (3)) and stiffness Q (see Eq. (6)) depend much on the SO β through the constants $\mathcal{J}_-$ in Eq. (3) and $g(w_r)$ of the neutron skin τ (see Eq. (5)). In Eq. (6), ν is roughly constant ($\nu \approx 2 - 4$) for all Skyrme forces at $\beta = 0$ but significantly varies as function of β depending on different Skyrme forces. The values of ν are mostly smaller than 9/4 suggested in [1] (besides of SGII where we found much larger values). Swiatecki and his collaborators found [2] the stiffness $Q \approx 14 - 20$ MeV from fitting the nuclear IVGDR energies calculated in the simplest hydrodynamic model (HDM) to the experimental data. Then, larger values $Q \approx 30 - 35$ MeV were suggested in the last two references in [1]. In spite of the several misprints in these derivations [2] (see [11]) the final result for the IVGDR energy constant D is close to that for the asymptotically large values of Q ($3JA^{-1/3}/Q \ll 1$). The IVGDR energy constants D of HDM are roughly in good agreement with the well-known experimental value $D_{\exp} \approx 80$ MeV for heavy nuclei within a precision better or of the order of 10 %, as shown in [11] (see also [15 - 17, 19 - 21]). More precise $A^{-1/3}$ dependence of D with the finite values of Q seems to be beyond the accuracy of these HDM calculations even accounting more consistently for the ES motion because of several other reasons (structure of the IVGDR, curvature, quantum-shell and Coulomb effects in the low energy region) towards the realistic calculations based on the Skyrme HF approach, see larger $Q \approx 30-80$ MeV found in [6, 12]. With larger Q (see the Table) the fundamental parameter of the LDM expansion in [1], $(9J/4Q)A^{-1/3}$, is really small for $A \gtrsim 40$ and therefore, the results obtained by using this expansion are justified.

The isovector energy k_S and stiffness Q coefficients for several Skyrme forces [14]; ν is the constant of Eq. (6); τ/I is the neutron skin calculated by Eq. (5); the intervals of functions $D_n(A)$ and $\bar{D}(A) = (D_1S_1 + D_2S_2)/(S_1 + S_2)$ are related to $A \approx 60-210$

Calculated quantities	Skyrme forces								
	SkM*	SkM	SIII	SGII	RATP	SkP	T6	SLy5	SLy7
$\mathcal{C}_-$ MeV · fm^5	-4.79	-4.69	-5.59	-0.94	13.9	-20.2	0	-22.8	-13.4
β	-0.64	-0.69	-0.57	-0.54	-0.52	-0.37	-0.45	-0.58	-0.65
k_S, MeV	-0.77	-1.90	-0.52	-0.21	1.42	-1.93	0	-6.96	-6.32
ν	0.34	0.46	1.42	17.9	0.45	1.76	4.30	0.59	0.67
Q, MeV	398	234	2168	60998	-270	823	∞	87	109
τ/I	0.044	0.090	0.40	0.040	0.072	0.035	0	0.0019	0.048
D_1, MeV	75 - 82	75 - 76	49 - 106	76 - 77	87	50 - 122	86 - 88	64 - 91	63 - 92
S_1, %	93 - 98	85 - 96	57 - 92	95 - 99	70 - 90	65 - 98	100	58 - 77	53 - 88
D_2, MeV	50 - 88	51 - 82	118 - 79	51 - 81	55 - 89	75 - 80	60 - 59	92 - 63	92 - 71
S_2, %	7 - 2	5 - 4	43 - 8	5 - 1	30 - 10	35 - 2	0	42 - 29	47 - 12
$\bar{D}$, MeV	73 - 82	71 - 76	79 - 104	74 - 77	77 - 87	70 - 69	86 - 88	76 - 84	77 - 89

The Table shows the IVGDR energies $D_n = \hbar\omega_n A^{1/3}$ ($n = 1, 2$) and EWSR S_n (normalized to 100 % for both peaks) obtained within a more precised FLDM [17]. The IVGDRs even for the spherical nuclei have a double-resonance structure, the main peak $n = 1$ which exhausts mainly the EWSR for almost all Skyrme forces and the satellite one $n = 2$ with significantly smaller EWSR contribution proportional to the asymmetry parameter I, especially for heavy nuclei. The last row shows the average $\bar{D}$ weighted by their EWSR distribution in rather good agreement with the experimental data within the same accuracy about 10 %, including SLyb230 ($\bar{D} = 81-91$ MeV) and skipped here in Table 1 for the sake of space. Exclusion can be done for the Skyrme forces SIII (see the Table) and SLya230 ($\bar{D} = 101-105$ MeV) of [14]. Note that the main characteristics of the IVGDR described by $\bar{D}$ are almost insensitive to the isovector surface energy constant k_S.

As conclusions, simple solutions of the isovector particle density and energies, in the leading ES approximation were used for analytical calculations of the neutron skin and isovector stiffness coefficients. Results for the isovector surface energy constant k_S and stiffness Q are rather sensitive to the choice of the Skyrme force parameters, especially those in the isovector gradient terms ($\mathcal{C}_-$) and SO interaction (β). The mean IVGDR energies and sum rules calculated within the FLDM [17] for the most of constants k_S and Q of the Table are in good agreement with the experimental data. For further perspectives, it would be interesting to compare the found constants with those of [19 - 21] within the macroscopic-microscopic models accounting however for the critical comments mentioned above, especially concerning the structure of the IVGDR. We are going to analyze the k_S dependence of the IVGDR satellite within the FLDM in relation to the well-known pygmy GDR resonances [22, 23] which are expected to be more sensitive to the values of k_S. It would be also worth to apply our results to calculations of the energies and sum rules for the isovector low-lying collective states within the periodic orbit theory [13, 24 - 26].

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ЯДЕРНА ЕНЕРГІЯ АСИМЕТРІЇ, НЕЙТРОННА ШУБА ТА ІЗОВЕКТОРНА ЖОРСТКІСТЬ

Ізовекторна густина частинок і коефіцієнт поверхневого натягу для середньої енергії зв'язку в наближенні різкого краю протонно-нейтронного асиметричного ядра використовуються для аналітичних розрахунків його нейтронної шуби та коефіцієнтів ізовекторної жорсткості. Результати значно відрізняються від відомих величин для більшості сил Скірма. Енергії та правила сум для ізовекторних дипольних гігантських резонансів, отриманих у рамках фермі-ріднннокрапельної моделі ядра, добре узгоджуються з експериментальними даними.

Ключові слова: ядерна енергія зв'язку, поверхнева енергія симетрії, протон-нейтронна асиметрія, нейтронна шуба, ізовекторна жорсткість, дипольні гігантські резонанси.

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ЯДЕРНАЯ ЭНЕРГИЯ АСИММЕТРИИ, НЕЙТРОННАЯ ШУБА И ИЗОВЕКТОРНАЯ ЖЕСТКОСТЬ

Изовекторная плотность частиц и коэффициент поверхностного натяжения для средней энергии связи в приближении резкого края протонно-нейтронного асимметричного ядра используются для аналитических расчетов его нейтронной шубы и коэффициентов изовекторной жесткости. Результаты значительно отличаются от известных величин для большинства сил Скирма. Энергии и правила сумм для изовекторных дипольных гигантских резонансов, полученных в рамках ферми-жидкокапельной модели ядра, хорошо согласуются с экспериментальными данными.

Ключевые слова: ядерная энергия связи, поверхностная энергия симметрии, протон-нейтронная асимметрия, нейтронная шуба, изовекторная жесткость, дипольные гигантские резонансы.

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