ЯДЕРНА ФІЗИКА

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DESCRIPTION OF NUCLEON TRANSFERS PROCESSES BY A COUPLED CHANNEL METHOD WITH TWO-CENTER STATES

The problem of quantum description of near-barrier fusion of heavy nuclei taking place under strong coupling of relative motion with external nucleons transfers is studied. The method of perturbed stationary states, founded on decomposing of a full wave function of a system of two nuclei and nucleon by a system of two-center nucleon wave functions, is applied for the description of nucleons transfers at low-energy nuclear reactions. The two-center nucleon energy levels – additions to nucleus-nucleus potential in a channels, and wave functions are calculated by a numerical solution of a Schrödinger equation for an arbitrary axial-symmetrical field with spin-orbit interactions, based on decomposing on Bessel functions and difference scheme along internuclear axis.

Keywords: nuclear fusion reactions, nucleons transfers, two-center nucleons states, coupled channel method.

Introduction

The reactions with neutron-rich nuclei have recently received increased interest as well experimentally as theoretically. The possibility to perform experiments with neutron-rich radioactive fission fragments opens new doors for the production and study of new isotopes and, probably, for synthesis of new superheavy elements [1, 2]. Also great efforts have been devoted to studying near-barrier fusion of light weakly bound nuclei [2, 3]. Unusual effects are expected here both from the halo structure of these nuclei and from the specific tunneling mechanism of the composed weekly bound system that is of general interest for quantum theory. Neutron transfer cross sections are known to be rather large at near-barrier energies of heavy-ion collisions – the result of significant extension of the wave functions of neutrons from the outer nuclear shells. A significant increase in fusion cross sections is observed in a series of reactions if nuclei with excessive neutrons participate in them. Such behavior has established in particular for two low energy fusion reactions, ^{18}O + ^{58}Ni and ^{16}O + ^{60}Ni [4], with similar compound nuclei (Fig. 1). In [2], the transition of external neutrons from the level of the ^{18}O nucleus to underlying levels of the ^{58}Ni nucleus was referred to as the fundamental source of additional energy in the translational motion of nuclei, raising the possibility of overcoming the Coulomb barrier. The interrelated processes of the generation of molecular states in the two nucleus ^{18}O + ^{58}Ni system, the transitions between such levels in the process of nuclei collision, and the transfer of neutrons between nuclei were investigated on time-dependent quantum description [5].

As a consequence there is a prevailing view that coupling with the transfer channels should play an

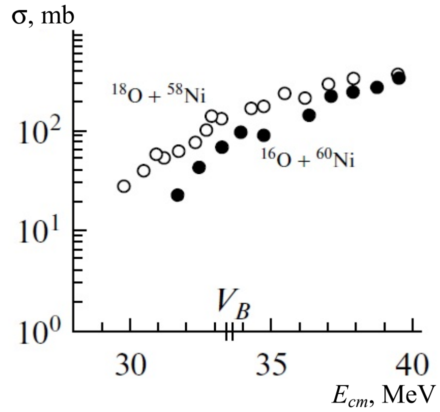


Fig. 1. Experimental dependence of nucleus fusion cross section σ in reactions ^{18}O + ^{58}Ni (white dots) and ^{16}O + ^{60}Ni (black dots) on the energy in the system at the center of mass E_{cm}, according to [4]; V_B is the Coulomb barrier height.

important role in sub-barrier fusion of heavy nuclei [3, 6]. However, if an influence of collective excitations (rotation of deformed nuclei and surface vibrations) on near-barrier fusion of heavy nuclei is well studied experimentally and well understood theoretically, the role of neutron transfer is not so clear. It is very difficult, for many reasons, to take into account explicitly the transfer channels within a consistent channel coupling approach used successfully for the description of collective excitations in the near-barrier fusion processes [7, 8]. In early analysis of nucleons transfer processes coupled channel matrix was used only in phenomenological model [9]. Fundamental coupled channel equations for reactions with particles redistribution were formulated in [10, 11]. In the present study, the correct channel equations for fusion and neutrons transfers channels at head-on collision in reaction ^{18}O + ^{58}Ni are formulated.

Theory

The microscopic description of capture of nuclei (with masses m_1, m_2) and external neutron (with mass m_3) transfers guesses the solution of a multi-dimensional stationary Schrödinger equation with Hamiltonian

$$H = -\frac{\hbar^2}{2m_1}\Delta_{\vec{r}_1} - \frac{\hbar^2}{2m_2}\Delta_{\vec{r}_2} + V_{12}(R) - \frac{\hbar^2}{2m_3}\Delta_{\vec{r}_3} +$$

$$+ V_{13}(\rho_{13}) + \hat{V}_{LS,13}(\vec{\rho}_{13}) + V_{23}(\rho_{23}) + \hat{V}_{LS,23}(\vec{\rho}_{23}), \quad (1)$$

with central Wood's - Sakson's potentials

$$V_{13}(\rho_{13}),\ V_{23}(\rho_{23}),$$

$$\vec{R} = \vec{r}_2 - \vec{r}_1,\ \vec{\rho}_1 = \vec{r}_3 - \vec{r}_1,\ \vec{\rho}_2 = \vec{r}_3 - \vec{r}_2, \quad (2)$$

and spin-orbit interactions

$$\hat{V}_{LS,13} = -\frac{b}{2\hbar}\vec{\sigma}\left[(\nabla_{\vec{\rho}_{13}} V_{13})\boldsymbol{p}_3\right],$$

$$\hat{V}_{LS,23} = -\frac{b}{2\hbar}\vec{\sigma}\left[(\nabla_{\vec{\rho}_{23}} V_{23})\boldsymbol{p}_3\right], \quad (3)$$

inclusive Pauli matrices $\vec{\sigma} = \{\sigma_x, \sigma_y, \sigma_z\}$ and neutron momentum operator $\boldsymbol{p}_3$. Application of the Jacobi coordinates

$$\vec{r} = \vec{r}_3 - \vec{r}_A,\ \vec{r}_A = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}, \quad (4)$$

result in the following Hamiltonian at center of mass system

$$H = -\frac{\hbar^2}{2M}\Delta_{\vec{R}} - \frac{\hbar^2}{2\mu}\Delta_{\vec{r}} + V_{12}(R) + V_{13}(\rho_{13}) +$$

$$+ \hat{V}_{LS,13}(\vec{\rho}_{13}) + V_{23}(\rho_{23}) + \hat{V}_{LS,23}(\vec{\rho}_{23}), \quad (5)$$

$$1/M = 1/m_1 + 1/m_2,\ 1/\mu = 1/m_3 + 1/(m_1 + m_2). \quad (6)$$

In work [11] for a system of three spinless particles (two nuclei and an electron) in the adiabatic basis a one-dimensional system of differential equations is obtained that represents the relative motion of the nuclei with account of the Coriolis interaction of the electron and nuclear motions. It is well known, that for heavy ion fusion processes head-on collisions play most important role. For such collisions mean angular velocity of nuclei is small and we may neglect to Coriolis interaction.

The coupled channel method [8, 10] based on wave function eigenfunctions expansion for internal degrees of freedom. For neutrons from the outer nuclear shells this eigenfunctions may be calculated in two center shell model with stationary Schrödinger equation

$$\left[-\frac{\hbar^2}{2\mu}\Delta_{\vec{r}} + V(\vec{r}) + \hat{V}_{LS}(\vec{r})\right]\Phi_\alpha(\vec{r};\vec{R}) = \varepsilon_\alpha(R)\Phi_\alpha(\vec{r};\vec{R}). \quad (7)$$

Wave functions

$$\Phi_\alpha(\vec{r};\vec{R}) = \begin{pmatrix}\psi_{\alpha1}(\vec{r};\vec{R})\\ \psi_{\alpha2}(\vec{r};\vec{R})\end{pmatrix}, \quad (8)$$

and energy levels $\varepsilon_\alpha(R)$ for internuclear distance R are calculated by a numerical solution of a Schrödinger's equations at cylindrical coordinates (ρ, φ, z) [12, 13]

$$\left[-\frac{\hbar^2}{2m}\Delta + V(\rho,z) + i\frac{b}{2}\frac{1}{\rho}V_\rho\frac{\partial}{\partial\varphi}\right]\psi_{\alpha1} +$$

$$+ i\frac{b}{2}e^{-i\varphi}\left[i\left(V_\rho\frac{\partial}{\partial z} - V_z\frac{\partial}{\partial\rho}\right) - \frac{1}{\rho}V_z\frac{\partial}{\partial\varphi}\right]\psi_{\alpha2} = \varepsilon_\alpha\psi_{\alpha1}, \quad (9)$$

$$\left[-\frac{\hbar^2}{2m}\Delta + V(\rho,z) - i\frac{b}{2}\frac{1}{\rho}V_\rho\frac{\partial}{\partial\varphi}\right]\psi_{\alpha2} -$$

$$- i\frac{b}{2}e^{i\varphi}\left[i\left(V_\rho\frac{\partial}{\partial z} - \tilde{V}_z\frac{\partial}{\partial\rho}\right) + \frac{1}{\rho}V_z\frac{\partial}{\partial\varphi}\right]\psi_{\alpha1} = \varepsilon_\alpha\psi_{\alpha2}, \quad (10)$$

with designation $V_\rho = \frac{\partial}{\partial\rho}V$, $V_z = \frac{\partial}{\partial z}V$. Quantum number α includes total angular momentum projection on internuclear axis m_j and number n of energy level, $\varepsilon_\alpha(R) = \varepsilon_{n\Omega}(R)$, $\Omega = |m_j|$.

Wave function of all system may be decomposed by a system of two-center nucleon wave functions

$$\Psi(\vec{r};\vec{R}) = \sum_\alpha F_\alpha(\vec{R})\Phi_\alpha(\vec{r};\vec{R}), \quad (11)$$

with added expansion by partial waves

$$F_\alpha(\vec{R}) = \sum_{LM} i^L\sqrt{4\pi(2L+1)}\frac{F_{\alpha LM}(R)}{R}Y_{LM}(\Omega). \quad (12)$$

This method results in coupled channel equations for function $F_{\alpha LM}(R)$ [10]

$$\left[-\frac{\hbar^2}{2M}\frac{d^2}{dR^2}+\frac{\hbar^2 L(L+1)}{2MR^2}+\varepsilon_\alpha(R)-E+V_{12}(R)\right]F_{\alpha LM}(R)+$$

$$+\sum_{\alpha' L' M'}\left\{\frac{\hbar^2}{M}Q_{\alpha LM\alpha' L' M'}(R)\frac{dF_{\alpha' L' M'}}{dR}+\frac{\hbar^2}{2M}K_{\alpha LM\alpha' L' M'}(R)F_{\alpha' L' M'}(R)\right\}=0, \tag{13}$$

$$Q_{\alpha LM\alpha' L' M'}(R)=i^{L'-L}\sqrt{\frac{(2L'+1)}{(2L+1)}}\int \bar{Y}_{LM}(\Omega)\langle\Phi_\alpha(\vec{R},\vec{r})|\nabla_{\vec{R}}|\Phi_{\alpha'}(\vec{R},\vec{r})\rangle Y_{L'M'}(\Omega)d\Omega\,, \tag{14}$$

$$K_{\alpha LM\alpha' L' M'}(R)=i^{L'-L}\sqrt{\frac{(2L'+1)}{(2L+1)}}\int \bar{Y}_{LM}(\Omega)\langle\Phi_\alpha(\vec{R},\vec{r})|\Delta_{\vec{R}}|\Phi_{\alpha'}(\vec{R},\vec{r})\rangle Y_{L'M'}(\Omega)d\Omega\,. \tag{15}$$

For primitive model of spherically symmetric matrices for head-on collisions and fusion processes

$$Q_{\alpha LM\alpha' L' M'}(R)=Q_{\alpha\alpha'}(R)\delta_{LL'}\delta_{MM'}\,,$$

$$K_{\alpha LM\alpha' L' M'}(R)=K_{\alpha\alpha'}(R)\delta_{LL'}\delta_{MM'}\,, \tag{16}$$

formulas (14), (15) are simplified

$$Q_{\alpha\alpha'}(R)=-\langle\Phi_\alpha(R,\vec{r})|\frac{\partial}{\partial R}|\Phi_{\alpha'}(R,\vec{r})\rangle\,, \tag{17}$$

$$K_{\alpha\alpha'}(R)=-\frac{2}{R}\langle\Phi_\alpha(R,\vec{r})|\frac{\partial}{\partial R}|\Phi_{\alpha'}(R,\vec{r})\rangle-$$

$$-\langle\Phi_\alpha(R,\vec{r})|\frac{\partial^2}{\partial R^2}|\Phi_{\alpha'}(R,\vec{r})\rangle\,. \tag{18}$$

Matrices elements with relating to same nucleus α, α' states generally speaking don't vanish in limit $R\to\infty$ [10] and it is serious problem. For purpose to correct apply formulas (12) - (18) with renormalized matrices Q, K for nucleon transfer description we begin from analyzing time dependent task. Nucleon transfer from nucleus 1 to nucleus 2 or fusion at nuclear reaction are described by Schrödinger equation

$$i\hbar\frac{\partial}{\partial t}\Psi(\vec{r};\vec{R})=H\Psi(\vec{r};\vec{R})\,, \tag{19}$$

with initial condition is

$$\Psi(\vec{r},\vec{R},t=0)=\phi_n(\vec{r}_3-\vec{r}_1)\varphi_{\vec{k}}(\vec{r}_A-\vec{r}_2)\,, \tag{20}$$

where $\phi_n(\vec{r}_3-\vec{r}_1)$ − nucleon bound state wave function and $\varphi_{\vec{k}}(\vec{r}_A-\vec{r}_2)$ − wave packet function of relative motion nucleus 1 together with nucleon and nucleus 2 with mean momentum $\hbar\vec{k}$. In the extreme case $V\to V_{13}(\rho_1)$ − wave packet (20) will free move in the $\vec{k}$ direction. Such property is nature of first order transfer differential equations in partial derivatives [14]. Therefore the main importance of all $Q_{\alpha\alpha'}(R)\frac{d}{dR}$ items and $K_{\alpha\alpha'}(R)$ items with α, α' states, relating to same nucleus in $R\to\infty$ limit, is providing with wave packet move and one particle excitations without nucleons transfer. For description only nucleon transfers between nuclei conditioned by transition between two-center states at nucleus-nucleus collision and fusion we may reserve in equations (13) only symmetric items with α, α' states appurtenant to the different nuclei

$$T_{\alpha\alpha'}(R)=T_{\alpha'\alpha}(R)=\int\left[\frac{\partial}{\partial R}\Phi^*_\alpha(R,\vec{r})\right]\times$$

$$\times\left[\frac{\partial}{\partial R}\Phi_{\alpha'}(R,\vec{r})\right]d\vec{r}\,. \tag{21}$$

The two-center nucleon wave functions phases must ensure continuity of wave functions $\Phi_\alpha(R,\vec{r})$ and its derivates $\frac{\partial}{\partial R}\Phi_\alpha(R,\vec{r})$. Resulting partial waves method application result in ordinary differential coupled channel equation for radial functions $F_\alpha(R)$

$$\left[-\frac{\hbar^2}{2M}\frac{d^2}{dR^2}+\frac{\hbar^2 L(L+1)}{2MR^2}+\varepsilon_\alpha(R)-E+V_{12}(R)\right]F_\alpha(R)=\sum_{\alpha'}\frac{\hbar^2\lambda}{2M}\tilde{T}_{\alpha\alpha'}(R)F_{\alpha'}(R)\,, \tag{22}$$

where $\lambda\sim 1$ − some renormalized factor. Boundary conditions for equation $H\Psi(\vec{r};\vec{R})=E\Psi(\vec{r};\vec{R})$ are renormalized to standard boundary condition for partial waves. Now equations (21), (22) give simple microscopic model validation for head-on collisions and fusion processes. After transformation set of differential equations and boundary conditions into difference equations and boundary conditions we receive the linear equation set and can solve it by famous numerical methods, for example, as for collective degrees of freedom [7 - 9].

Results and discussion

The results of calculation energy levels $\varepsilon_\alpha(R)=\varepsilon_{n\Omega}(R)$ for spherical nuclei $^{18}O + {}^{58}Ni$ system are shown in Fig. 2.

Some probability densities

$$p(\rho, z) = |\psi_{\alpha 1}(\vec{r})|^2 + |\psi_{\alpha 2}(\vec{r})|^2, \tag{23}$$

for spherical nuclei $^{18}O + {}^{58}Ni$ system are shown in Figs. 3 - 5. Some coupled channel matrices (21) elements are shown in Fig. 6.

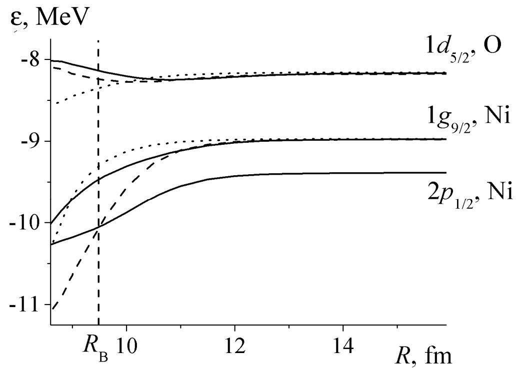


Fig. 2. Some two-center neutron energy levels for a system $^{18}O + {}^{58}Ni$ as a function of internuclear distance R for total angular momentum projection on internuclear axis $\Omega = 1/2$ (solid lines), $\Omega = 3/2$ (dashed lines), $\Omega = 5/2$ (dotted lines), R_B corresponds to top of a Coulomb barrier.

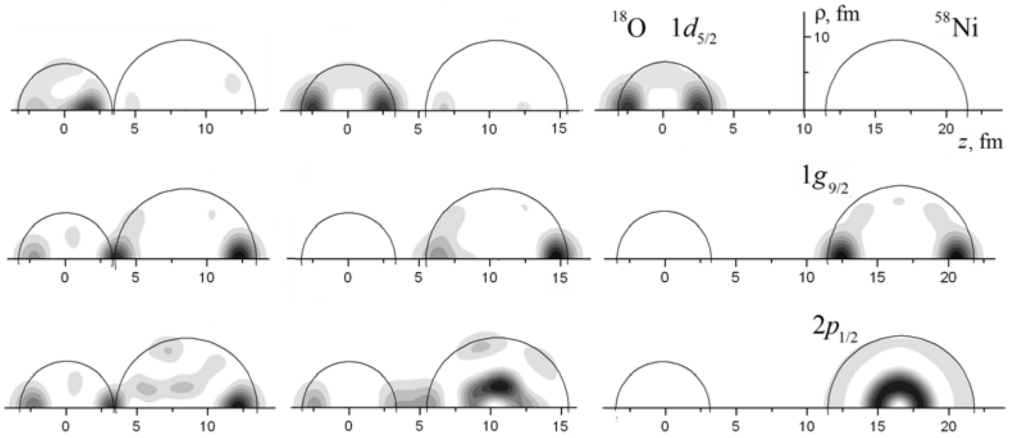


Fig. 3. Some two-center neutron wave functions for a system $^{18}O + {}^{58}Ni$ at three values of internuclear distance R for total angular momentum projection on internuclear axis $\Omega = 1/2$.

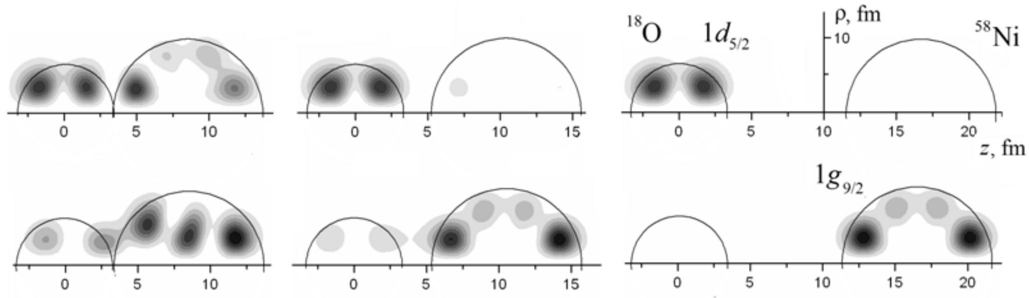


Fig. 4. Some two-center neutron wave functions for a system $^{18}O + {}^{58}Ni$ at three values of internuclear distance R for total angular momentum projection on internuclear axis $\Omega = 3/2$.

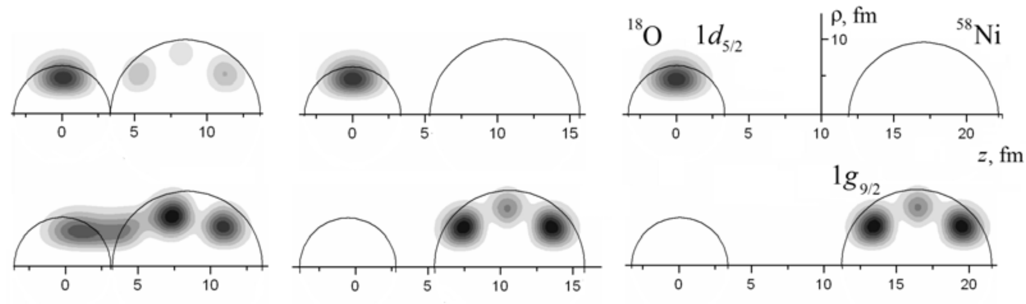
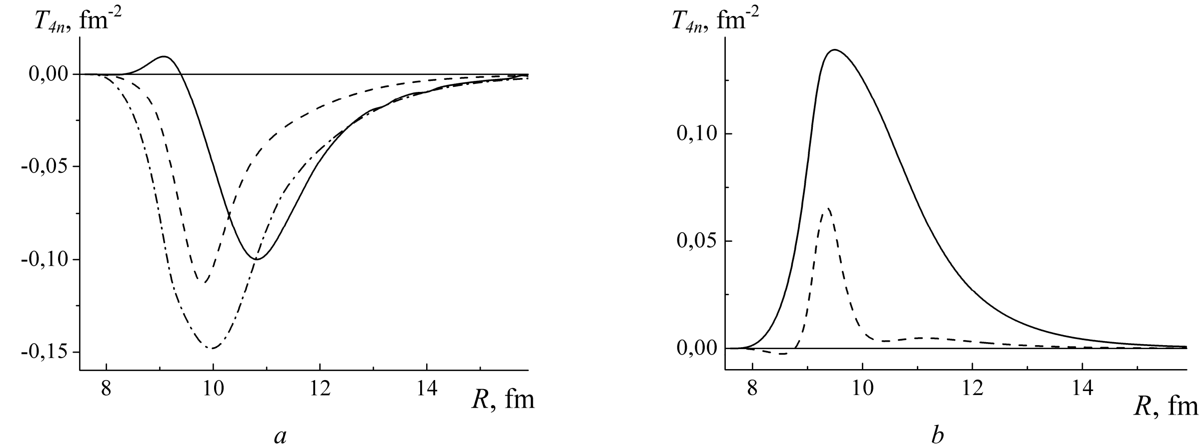


Fig. 5. Some two-center neutron wave functions for a system $^{18}O + {}^{58}Ni$ at three values of internuclear distance R for total angular momentum projection on internuclear axis $\Omega = 5/2$.



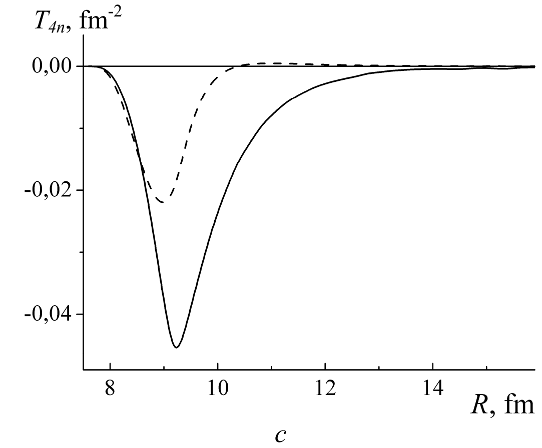


Fig. 6. Coupled channel matrices (22) T_{43} (solid lines), T_{42} (dash-dotted line) and T_{41} (dashed lines) for neutron states *1* − $1f_{5/2}$; *2* − $2p_{1/2}$; *3* − $1g_{9/2}$ of ^{58}Ni; *4* − $1d_{5/2}$ of ^{18}O for total angular momentum projection on internuclear axis $\Omega = 1/2$ (*a*), $\Omega = 3/2$ (*b*) and $\Omega = 5/2$ (*c*).

During internuclear distance R reducing, potential barrier between the potential wells of colliding nuclei is changing slowly reduce. As the nuclei approach one another, the barrier height falls and large values of non-diagonal coupled matrices (21) result in neutron transfer from upper level $1d_{5/2}$ of ^{18}O to lower free levels $1g_{9/2}$, $2p_{1/2}$, $1f_{5/2}$ of ^{58}Ni with increasing of probability Coulomb barrier for nuclear fusion.

Conclusion

The proposed model for calculating nucleon states in asymmetric nuclear systems has made it possible to explain qualitatively experimental data on the excess of the cross section for fusion in the ^{18}O + ^{58}Ni reaction above the cross section for fusion in the ^{16}O + ^{60}Ni reaction. This model can be generalized for few independent nucleons (neutrons and protons) in two-center shell model with few Q-values of reaction.

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ОПИС ПРОЦЕСІВ НУКЛОННИХ ПЕРЕДАЧ МЕТОДОМ СИЛЬНОГО ЗВ'ЯЗКУ КАНАЛІВ ІЗ ДВОЦЕНТРОВИМІ СТАНАМИ

Вивчена проблема квантового опису білябар'єрного злиття важких ядер в умовах сильного зв'язку відносного руху з передачами зовнішніх нуклонів. Для опису нуклонних передач при низькоенергетичних ядерних реакціях застосовано метод збурених стаціонарних станів, заснований на розкладанні повної хвильової функції системи двох ядер і нуклона за системою двоцентрових хвильових функцій. Двоцентрові нуклонні рівні енергії – добавки до ядро-ядерного потенціалу в каналах – та хвильові функції знайдено шляхом чисельного розв'язку рівняння Шредінгера для довільного аксіально-симетричного поля, заснованого на розкладанні за функціями Бесселя і різницевої схемою вздовж меж'ядерної осі.

Ключові слова: ядерні реакції злиття, нуклонні передачі, двоцентрові нуклонні стани, метод сильного зв'язку каналів.

В. В. Самарин

ОПИСАНИЕ ПРОЦЕССОВ НУКЛОННЫХ ПЕРЕДАЧ МЕТОДОМ СИЛЬНОЙ СВЯЗИ КАНАЛОВ С ДВУХЦЕНТРОВЫМИ СОСТОЯНИЯМИ

Изучена проблема квантового описания околобарьерного слияния тяжелых ядер в условиях сильной связи относительного движения с передачами внешних нуклонов. Для описания нуклонных передач при низкоэнергетических ядерных реакциях применен метод возмущенных стационарных состояний, основанный на разложении полной волновой функции системы двух ядер и нуклона по системе двухцентровых волновых функций. Двухцентровые нуклонные уровни энергии – добавки к ядро-ядерному потенциалу в каналах – и волновые функции найдены путем численного решения уравнения Шредингера для произвольного аксиально-симметричного поля, основанного на разложении по функциям Бесселя и разностной схеме вдоль межъядерной оси.

Ключевые слова: ядерные реакции слияния, нуклонные передачи, двуцентровые нуклонные состояния, метод сильной связи каналов.

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