УДК 539.17.01

# = ЯДЕРНА ФІЗИКА =

#### A. V. Mikhailov<sup>1</sup>, Yu. N. Pavlenko<sup>2</sup>, V. L. Shablov<sup>1</sup>, A. V. Stepaniuk<sup>2</sup>, I. A. Tyras<sup>1</sup>

 <sup>1</sup> Obninsk Institute for Nuclear Power Engineering, National Research Nuclear University MePHI, Obninsk, Russia
 <sup>2</sup> Institute for Nuclear Research, National Academy of Sciences of Ukraine, Kyiv

# COULOMB INTERACTION EFFECTS IN MANY-PARTICLE NUCLEAR REACTIONS WITH TWO-FRAGMENT RESONANCE FORMATION

The modified final-state interaction theory taking into consideration the Coulomb interaction between two-fragment nuclear resonance decay products and accompanying reaction products is developed including the case of near-threshold resonances. The branching ratio change is also studied for the near-threshold resonance <sup>7</sup>Li<sup>\*</sup>( $E_x = 7.45$  MeV), which is formed in the reaction <sup>7</sup>Li( $\alpha$ ,  $\alpha$ )<sup>7</sup>Li<sup>\*</sup> at  $E_\alpha = 27.2$  MeV.

*Keywords:* three-particle reactions, nuclear resonances, resonance theory, Coulomb interaction, near-threshold resonances, decay channels, branching ratio.

#### Introduction

In this article we study the possible deviations (with respect to the properties of the resonance in the isolated pair) of the parameters of a two-body resonance in the Coulomb field of a third particle, especially in the case of reactions with the nearthreshold resonance formation. The reactions of the type

$$a + A \rightarrow 1 + b^* \rightarrow 1 + 2 + 3$$

have been extensively investigated lately in a number of both theoretical and experimental studies [1 - 9]. The influence of accompanying particle on the resonance decay is known as the PSI (postcollision interaction) effect [2, 8]. This influence is most pronounced in the cases when the reaction final state is characterized by the great values of Coulomb parameters, which determines the external Coulomb field intensity.

The experimental data obtained in the reactions with light nuclei resonant state excitation have shown that the deviation pointed out could range up to 100 % values for the observable resonance excitation energy  $E_R^*$  and its half width  $\Gamma^*/2$  with respect to the parameters  $E_R$  and  $\Gamma/2$ , determining resonance complex the isolated energy  $Z_R = E_R - i\Gamma/2$ . It was shown in [4, 10] that in the case of resonances far from the decay thresholds the resonance curves are always broadened in accordance with experimental data. The case of the near-threshold resonance is more complicated: the resonance peak can be narrowed at some kinematical conditions [7]. Moreover, the effect of the branching ratio change can take place [4, 11].

The modification of the theory represented in [4, 10] is developed below for the case of the postcollision Coulomb interaction in reactions with the near-threshold resonance formation.

#### The model

In the case under investigation short range nuclear forces are responsible for the resonance formation, so the known expression for the reaction amplitude, which takes into account the Coulomb interaction of the reaction products on the background of their nuclear interaction, can be used [10]:

$$T(k_{23}\vec{p}_{1},\vec{p}_{0},E+i0) = T_{C}\left(k_{23}\vec{p}_{1},\vec{p}_{0},E+i0\right) + \left\langle \psi_{\beta}^{-}(\vec{k}_{23}\vec{p}_{1}) \middle| V^{\beta} - U_{\beta} + (V^{\beta} - U_{\beta})G(E+i0)(V^{\alpha} - U_{\alpha}) \middle| \psi_{\alpha}^{+}(\vec{p}_{0}) \right\rangle.$$
(1)

Here, indexes  $\alpha$  and  $\beta$  denote the initial and final reaction channels respectively, G(Z) = $= (Z - H)^{-1}$  is the total Green's function of the system with the Hamiltonian H, Z = E + i0 is the energy of the system. The potential  $V^{\beta} = V_s^{\beta} + V_c^{\beta}$  is the sum of the nuclear and Coulomb potentials, acting between particles from different fragments in the final channel,  $U_{\beta}$  is the Coulomb interaction between produced fragments,  $\left| \psi_{\beta}^{-} (\vec{k}_{23} \vec{p}_1) \right\rangle$  is the wave function of the outgoing reaction channel of the form

$$\left| \Psi_{\beta}^{-} \left( \vec{k}_{23} \vec{p}_{1} \right) \right\rangle = \left| \Psi_{C}^{-} \left( \vec{k}_{23} \vec{p}_{1} \right) \right\rangle \left| \prod_{j=1}^{3} \Phi_{j} \right\rangle,$$

where  $|\Phi_j\rangle$  is the bound-state wave function of the fragment *j* and  $|\psi_c(\vec{k}_{23}\vec{p}_1)\rangle$  is the wave function of the pure Coulomb scattering of produced fragments,

<sup>©</sup> A. V. Mikhailov, Yu. N. Pavlenko, V. L. Shablov, A. V. Stepaniuk, I. A. Tyras, 2014

i.e. for the potential  $U_{\beta}$ . The state  $|\psi_{\alpha}^{+}(\vec{p}_{0})\rangle$  is determined in the same way. The channel Hamiltonian  $H_{\beta}$  has the form

$$H_{\beta} = H_0 + \sum_j h_j,$$

where  $H_0$  is the free Hamiltonian of the three-body system and  $h_j$  denotes the *j*-th fragment internal motion, so that

$$h_j \left| \Phi_j \right\rangle = -\chi_j^2 \left| \Phi_j \right\rangle,$$

 $-\chi_j^2$  being the binding energy of the fragment j. Usually,  $\vec{k}_{23}$  and  $\vec{p}_1$  are the Jacobi coordinates of the three-body system in the momentum space, therefore, the energy of the system is equal to

$$E = \frac{k_{23}^2}{2\mu_{23}} + \frac{p_1^2}{2n_1} - \sum_{j=1}^3 \chi_j^2,$$

where  $\mu_{23}$  and  $n_1$  are the corresponding reduced masses

$$\mu_{23} = \frac{m_2 m_3}{m_2 + m_3} \qquad n_1 = n_{1,23} = \frac{m_1 (m_2 + m_3)}{m_1 + m_2 + m_3}.$$

Finally,  $T_c$  is the amplitude of the pure Coulomb transition between channels  $\alpha$  and  $\beta$ .

To extract the resonant behavior of the reaction amplitude (1) we perform the following. We start

from the second resolvent identity for G(Z):

$$G(Z) = G_{23}(Z) + G_{23}(Z)V^{23}G(Z), \qquad (2)$$

where the operator  $G_{23}(Z) = (Z - H_{23})^{-1}$  is the Green's function for the Hamiltonian  $H_{23} = H_{\beta} + V_{23}$ . The operator  $V_{23}$  is the sum of the Coulomb and nuclear potentials acting between fragments 2 and 3, whereas  $V^{23}$  equals  $H - H_{23}$ . For further consideration the Hamiltonian  $H_{23}$  is conveniently represented in the form  $H_{23} = h_{23} + H_{01,23} + \sum_{j=1}^{3} h_j$ ,

where  $h_{23}$  is the Hamiltonian of the internal motion in the pair 23,  $H_{01,23}$  is the kinetic energy operator of the particle 1 and the pair 23 relative motion, so that in the momentum space  $H_{01,23}$  is the operator of multiplication by the value  $\frac{p_1^2}{2n}$ .

The operator  $G_{23}(Z)$  can be represented as

$$G_{23}(Z) = g_{23}\left(Z - H_{01,23} - \sum_{j=1}^{3} h_j\right)^{-1}$$
, where  $g_{23}(Z) =$ 

 $=(Z - h_{23})^{-1}$  is the Green's function of the pair 23. In its turn  $g_{23}(Z)$  is written in the form of the formal resonance theory expansion [10, 15]

$$g_{23}(Z) = R_{23}(Z) + \sum_{M} \left[ I + R_{23}(Z) W \right] \left[ \Phi_{M} \right] \frac{1}{\omega(Z)} \left\{ \Phi_{M} \left[ WR_{23}(Z) + I \right] \right\}.$$
(3)

In the expansion (3) it is supposed that the Hamiltonian  $h_{23}$  is represented in the form

$$h_{23} = \tilde{h}_{23} + W,$$

where the Hamiltonian  $\tilde{h}_{23}$  has the bound state embedded in the continuous spectrum,  $P = \sum_{M} |\Phi_{M}\rangle \langle \Phi_{M}|$  is the projection operator on this bound state, while W is some perturbation potential. The operator  $R_{23}(Z)$  is the resolvent of the Hamiltonian  $Qh_{23}Q$  in the truncated Hilbert space with Q = I - P

$$R_{23}(Z) = (ZQ - Qh_{23}Q)^{-1}Q.$$

The function  $\omega(Z)$  is determined as

$$\omega(Z) = Z - \varepsilon_0 - \langle \Phi_M | W + W R_{23}(Z) W | \Phi_M \rangle, \quad (4)$$

where  $\varepsilon_0$  is given by the relation

$$\tilde{h}_{23} | \Phi_{M} \rangle = \varepsilon_{0} | \Phi_{M} \rangle.$$

The expansion (3) is written on the assumption that the Hamiltonian  $h_{23}$  and  $\tilde{h}_{23}$  are invariant under space rotations, so that the index *M* in Eq. (4) can be arbitrary  $(-L \le M \le L, L)$  is the resonance angular momentum). In the case in question the function  $\omega(Z)$  can be represented as

$$\omega(Z) = \omega_0(Z)(Z - Z_R), \qquad (5)$$

where  $Z_R = E_R - i\frac{\Gamma}{2}$  is the energy of the resonance state. The explicit form of the function  $\omega_0(Z)$  can be found on the assumption that  $V_{23}$  and W are dilatation analytic potentials. Then the function  $\omega(Z)$  has a (many-sheeted) analytic continuation onto the part of the unphysical sheet by the law

$$\omega(Z) = \omega^{\theta}(Z) = \left\langle \Phi_{M}(\theta^{*}) \middle| W(\theta) + W(\theta)R^{\theta}(Z)W(\theta) \middle| \Phi_{M}(\theta) \right\rangle, \tag{6}$$

where

$$\left\langle \vec{r} \left| \Phi_{M} \left( \theta \right) \right\rangle = \left\langle \vec{r} \left| U \left( \theta \right) \right| \Phi_{M} \right\rangle = e^{\frac{3}{2} \theta} \Phi_{M} \left( e^{\theta} \vec{r} \right),$$
  
Im  $\theta > 0,$  (7)

 $U(\theta)$  being the dilatation operator,

$$W(\theta) = U(\theta)WU^{-1}(\theta),$$
  

$$R^{\theta}(Z) = U(\theta)R(Z)U^{-1}(\theta) =$$
  

$$= \left(ZQ(\theta) - Q(\theta)\tilde{h}_{23}(\theta)Q(\theta)\right)^{-1}Q(\theta) \qquad (8)$$

and so on (see, for example, [19] for details).

The resonance energy  $Z_R$  satisfies the equation

$$\omega^{\theta}(Z_R) = 0 \tag{9}$$

and

$$\omega_{0}(Z) = \frac{\omega^{\theta}(Z) - \omega^{\theta}(Z_{R})}{Z - Z_{R}} =$$

$$= 1 + \left\langle \Phi_{M}(\theta^{*}) \middle| W(\theta) R^{\theta}(Z) R^{\theta}(Z_{R}) W(\theta) \middle| \Phi_{M}(\theta) \right\rangle.$$
(10)

In the physical region (Z = E + i0) the function  $\omega(Z)$  can be represented in the equivalent form

$$\omega(E+i0) = A\left(E - \varepsilon_R + i\frac{\Gamma(E)}{2}\right), \quad (11)$$

where the value  $\varepsilon_R$  satisfies the equation

$$E - \varepsilon_{R} - \langle \Phi_{M} | W | \Phi_{M} \rangle - I(\varepsilon_{R} + i0) = 0,$$
  
$$I(Z) = \langle \Phi_{M} | WR(Z) W | \Phi_{M} \rangle$$
(12)

and

$$A = 1 - \frac{dI(E+i0)}{dE}\bigg|_{E=\varepsilon_R} > 0,$$

$$\Gamma(E) = -2A^{-1} \operatorname{Im} I(E+i0).$$
(13)

In case of the resonance far from decay thresholds the values  $\varepsilon_R$  and  $E_R$  are equal and  $\Gamma = \Gamma(E_R)$ , while in the case of near-threshold resonance the values  $\varepsilon_R$  and  $E_R$  may be different and the resonance width becomes energy dependent [10, 15, 16]

$$\Gamma(E) = \Gamma_1 + \Gamma_2(E). \tag{14}$$

For example, let us suppose that the resonance under investigation decays into the two charged fragments. In this case the threshold behavior of the width  $\Gamma_2(E)$  is described by the expression

$$\Gamma_{2}(E) = Bk_{23}^{L+1}e^{-\pi\eta_{23}}\left|\Gamma(L+1+i\eta_{23})\right|^{2},$$

where *B* is a constant and  $\eta_{23}$  is the Coulomb parameter of the pair 23:  $\eta_{23} = \frac{q_2 q_3 \mu_{23}}{R_{23}}$ ,  $q_i$  is the

charge of *i*-th fragment.

Substituting the expansion (3) in the second resolvent identity (2), we find

$$G(Z) = \left[ R_{23}(Z_{23}) + \sum_{M} \left| \Psi_{LM}(Z_{23}) \right\rangle \frac{1}{\omega(Z_{23})} \left\langle \Psi_{LM}(Z_{23}) \right| \right] \times \left[ I + V^{23}G(Z) \right]$$
(15)

with 
$$Z_{23} = Z - H_{01,23} - \sum_{j=1}^{3} h_j$$
 and  $|\Psi_{LM}(Z_{23})\rangle = [I + R_{23}(Z_{23})W] |\Phi_{LM}\rangle.$ 

The equation (15) can be rearranged using the Veselova transformation [21] to extract the long-range part of the effective interaction potential between the resonance and the accompanying fragment as well as the resonant part of the total Green's function  $G_R(Z)$ .

From the equation (15) we have

$$G_{R}(Z) = \sum_{M} |\Psi_{LM}(Z_{23})\rangle \frac{1}{\omega(Z_{23})} \langle \Psi_{LM}(Z_{23}) | [I + V^{23}G(Z)].$$
(16)

Representing G(Z) as the sum

$$G(Z) = \tilde{G}(Z) + G_R(Z) \tag{17}$$

and  $G_R(Z)$  in the form

$$G_{R}(Z) = \sum_{M} \left| \Psi_{LM}(Z_{23}) \right\rangle \frac{1}{\omega(Z_{23})} B_{M}(Z), \quad (18)$$

we obtain that the kernel of the equation for the operator  $B_M(Z)$  is equal to

$$K_{MM'} = \left\langle \Psi_{LM} \left( Z_{23} \right) \middle| V^{23} \middle| \Psi_{LM'} \left( Z_{23} \right) \right\rangle \frac{1}{\omega(Z_{23})}.$$
 (19)

The operator 
$$\langle \Psi_{LM}(Z_{23}) | V^{23} | \Psi_{LM'}(Z_{23}) \rangle$$

describes the static part of the effective potential acting between the resonance and the accompanying particle. The long-range part of this potential in the momentum representation is written as

$$\frac{4\pi q_1 (q_2 + q_3)}{\left| \vec{p}_1 - \vec{p}_1 \right|^2} F_{MM'} (\vec{p}_1, \vec{p}_1), \qquad (20)$$

where  $F_{MM'}(\vec{p}_1, \vec{p}_1)$  is the form-factor of the unstable system

$$F_{MM'}\left(\vec{p}_{1},\vec{p}_{1}'\right) = \left(1 + \left\langle\Phi_{M}\right|WR^{2}\left(Z_{23} - \frac{p_{1}^{2}}{2n_{1}}\right)W\left|\Phi_{M}\right\rangle\right)\delta_{MM}$$

or after using the complex dilatation method

$$F_{MM'}\left(\vec{p}_{1},\vec{p}_{1}'\right) = \left(1 + \left\langle\Phi_{M}\left(\theta^{*}\right)\middle|W\left(\theta\right)R^{\theta}\left(Z_{23} - \frac{p_{1}^{2}}{2n_{1}}\right)\right)^{2} \times W\left(\theta\right)\middle|\Phi_{M}\left(\theta\right)\right\rangle\delta_{MM'}.$$

The last result shows that the expression  $F_{MM'}\left(\vec{p}_1, \vec{p}_1'\right) \omega_0^{-1} \left(Z_{23} - \frac{p_1^2}{2n_1}\right) \text{ differs from 1 by the}$ 

function which is proportional to  $Z_{23} - Z_R - \frac{p_1^2}{2n_1}$ , so that the kernel (15) can be represented as

$$K_{MM'} = K_{MM'}^C + \Delta K_{MM'}, \qquad (21)$$

where

$$K_{MM'}^{C}\left(\vec{p}_{1},\vec{p}_{1}'\right) = \left\langle \vec{p}_{1} \left| V_{1,23}^{C} \right| \vec{p}_{1}' \right\rangle \frac{1}{Z_{23} - Z_{R} - \frac{p_{1}^{2}}{2n_{1}}} \delta_{MM'}, \quad (22)$$

 $V_{1,23}^{C}$  being the pure Coulomb potential between the resonance and the third particle. The operator  $\Delta K_{MM'}$  describes the contribution in  $K_{MM'}$  of the short-range part of the effective potential and the non-resonant part of the kernel (15) (e.g. the part of  $K_{MM'}$ , which does not contain the resonance

propagator  $P_R(Z) = (Z_{23} - Z_R - H_{01,23})^{-1}$ ).

Introducing into consideration the Coulomb propagator  $P_R^c(Z)$ :

$$P_{R}^{c}(Z) = \left(Z_{23} - Z_{R} - H_{01,23} - V_{1,23}^{c}\right)^{-1}$$
(23)

we can rewrite the expression for  $G_R(Z)$  as

$$G_{R}(Z) = \sum_{M} |\Psi_{LM}(Z_{23})\rangle \frac{1}{\omega_{0}(Z_{23})} P_{R}^{c}(Z_{23}) \tilde{B}_{M}(Z)$$
(24)

with some operators  $\tilde{B}_{M}(Z)$  of a non-resonant type, the explicit forms of which are not essential for further consideration.

The following transformation of the expression (1) is based on the application of the effective charge method for the determination  $|\psi_c^-(\vec{k}_{23}, \vec{p}_1)\rangle$  [12, 13]:

$$\left| \Psi_{c}^{-}(\vec{k}_{23},\vec{p}_{1}) \right\rangle = \left| \Psi_{c\,23}^{-}(\vec{k}_{23}) \right\rangle \left| \Psi_{c\,1,23}^{-}(\vec{p}_{1}) \right\rangle + G^{c}(E+i0) \left( U_{\beta} - V_{23}^{c} - U_{1,23}^{c} \right) \left| \Psi_{c}^{-}(\vec{k}_{23},\vec{p}_{1}) \right\rangle.$$
(25)

In the expression (3)  $|\psi_{c23}(\vec{k}_{23})\rangle$  denotes the twobody Coulomb wave function for fragments 2 and 3,  $G^{c}(E+i0)$  is the Coulomb Green's function for reaction products. The potential  $U_{1,23}^{c}$  has the form:

$$U_{1,23}^{c}(\rho) = \frac{(\eta_{12} + \eta_{13})p_{1}}{n_{1}} \cdot \frac{1}{\rho_{1}},$$
 (26)

where  $\eta_{ij}$  is the Coulomb parameter for the pair ij;  $\vec{p}_1$  is the relative coordinate of the fragment 1 and the center of mass of the resonance  $b^*$ . Lastly, the wave function  $|\Psi_{c1,23}^-(\vec{p}_1)\rangle$  satisfies the following Schrödinger equation

$$\left(H_{01,23} + U_{1,23}^{c}(\rho_{1})\right)\left\langle\vec{\rho}|\psi_{c1,23}^{-}(\vec{p}_{1})\right\rangle = \frac{p_{1}^{2}}{2n_{1}}\left\langle\vec{\rho}_{1}|\psi_{c1,23}^{-}(\vec{p}_{1})\right\rangle.$$
(27)

Substituting the representation (17), (18) and (24) in the expression (1) for the reaction amplitude and using (25), we conclude that the last term in the right side of the expression (25) does not give the contribution in the resonant part of the amplitude (1). This resonant part is equal to

$$T_{R}(\vec{k}_{23}\vec{p}_{1},\vec{p}^{0},E+i0) = \sum_{M} \left\langle \psi_{c23}^{-}(\vec{k}_{23}) \middle| \left\langle \psi_{c1,23}^{-}(\vec{p}_{1}) \middle| \left\langle \Phi_{2} \middle| \left\langle \Phi_{3} \middle| \left( V_{23} - U_{23} \right) \middle| \Psi_{LM} \left( E^{C} + i0 - H_{01,23} \right) \right\rangle \right\rangle \times \frac{1}{\omega_{0} \left( E^{C} + i0 - H_{01,23} \right)} P_{R}^{c} \left( E^{C} - Z_{R} \right) \middle| C_{M} \left( \vec{p}^{0} \right) \right\rangle$$

$$\text{with } E^{C} = \frac{k_{23}^{2}}{2\mu_{23}} + \frac{p_{1}^{2}}{2n_{1}} \text{ and } \left| C_{M} \left( \vec{p}^{0} \right) \right\rangle = \left\langle \Phi_{1} \middle| \tilde{B}_{M} \left( V^{\alpha} - U_{\alpha} \right) \middle| \psi_{\alpha}^{+} \left( \vec{p}_{\alpha}^{0} \right) \right\rangle.$$

$$(28)$$

L

By virtue of the fact that the two-body Coulomb wave function in the momentum representation has a strong ( $\delta$ -function type) singularity in the forward

direction [10, 21] the expression (28) can be simplified to

$$T_{R}(\vec{k}_{23}\vec{p}_{1},\vec{p}_{0},E+i0) = \sum_{M} Y_{LM}(\vec{k}_{23})\chi_{23}(k_{23})\omega_{0}^{-1}\left(\frac{k_{23}^{2}}{2\mu_{23}}+i0\right)I_{M}(\vec{p}_{1},E+i0),$$
(29)

where  $\chi_{23}$  is the resonance decay vertex function

$$\chi_{23}(k_{23}) = \int d\Omega_{\vec{k}_{23}} Y_{LM}^*(\vec{k}_{23}) \left\langle \Psi_{c\,23}(\vec{k}_{23}) \middle| \left\langle \Phi_2 \middle| \left\langle \Phi_3 \middle| \left( V_{23} - U_{23} \right) \middle| \Psi_{LM} \left( \frac{k_{23}^2}{2\mu_{23}} + i0 \right) \right\rangle$$
(30)

and

$$I_{M}(\vec{p}_{1}, E+i0) = \left\langle \Psi_{c1,23}^{-}(\vec{p}_{1}) \middle| P_{R}^{c}(E^{C}-Z_{R}) \middle| C_{M}(\vec{p}^{0}) \right\rangle.$$
(31)

Taking into account the properties of integrals with the two-body Coulomb Green's function in the momentum representation [22], the non-resonant behavior of the functions  $\langle \vec{p}_1 | C_M(\vec{p}^0) \rangle$  and approximating for this reason  $\langle \vec{p}_1 | C_M(\vec{p}^0) \rangle$  by the proper constant  $\tilde{C}_M$ , we can rewrite the expression for  $I_M$  in the form

$$I_{M} = \int d\vec{p}_{1} \langle \psi_{c1,23}^{-}(\vec{p}_{1}) \Big| P_{R}^{c}(E^{C} - Z_{R}) \Big| \vec{p}_{1} \rangle \tilde{C}_{M},$$
$$\tilde{C}_{M} = \left\langle \frac{\vec{p}_{1}}{p_{1}} \sqrt{2n_{1}(E^{C} - E_{R})} \Big| C_{M} \left( \vec{p}^{0} \right) \right\rangle \quad (32)$$

or in the coordinate representation

$$I_{M} = (2\pi)^{\frac{3}{2}} \int d\vec{\rho}_{1} \langle \psi_{c1,23}^{-}(\vec{p}_{1}) | \vec{\rho}_{1} \rangle \langle \vec{\rho}_{1} | P_{R}^{c}(E^{C} - Z_{R}) | 0 \rangle \tilde{C}_{M}.$$
(33)

The matrix element  $\langle \vec{\rho}_1 | P_R^c (E^C - Z_R) | 0 \rangle$  is known in the explicit form [23]

$$\left\langle \vec{\rho}_{1} \right| P_{R}^{c} (E^{C} - Z_{R}) \left| 0 \right\rangle = -\frac{n_{1}}{2\pi\rho_{1}} \Gamma \left( 1 + i\nu \right) W_{-i\nu, \frac{1}{2}} \left( -2ik_{R}\rho_{1} \right),$$
(34)

where  $W_{\mu,\nu}(z)$  is the Whittaker function,  $k_R = \sqrt{2n_1(E^C - Z_R)}$  and  $\nu = \frac{q_1(q_2 + q_3)n_1}{k_R}$ . Using

the integral representation for this function and the Nordsieck formula [14]

$$\int d\vec{\rho} \left\langle \Psi_c^-(\vec{k}) \middle| \vec{\rho} \right\rangle \frac{e^{i\vec{k}\vec{\rho} - \mu\rho}}{\rho} =$$

$$=\frac{4\pi}{(2\pi)^{3/2}}e^{\frac{\pi}{2}\eta}\Gamma(1+i\eta)\frac{\left(\left(k'+i\mu\right)^{2}-k^{2}\right)^{i\eta}}{\left(\left(\bar{k}-\bar{k'}\right)^{2}+\mu^{2}\right)^{1+i\eta}}\qquad(35)$$

( $\eta$  is the corresponding two-body Coulomb parameter), we obtain after some transformations

$$I_{M} = \frac{4n_{1}k_{R}}{\left(p_{1} + k_{R}\right)^{3}}e^{-\frac{\pi}{2}\eta}\Gamma\left(1 + i\eta\right)\tilde{C}_{M}\sum_{j=1}^{4}A_{j}.$$
 (36)

The values  $A_i$  in (36) are defined by the relations

$$A_{1} = (1+i\eta) \int_{0}^{\infty} dx \ x^{i\nu} f(2+i\eta, 1-i\eta; x),$$

$$A_{2} = -(1+i\eta) \int_{1}^{\infty} dx \ x^{i\nu} f(2+i\eta, 1-i\eta; x),$$

$$A_{3} = -(1+i\eta) \int_{0}^{1} dx \ x^{i\nu+1} f(2+i\eta, 1-i\eta; x),$$

$$A_{4} = (1-i\eta) \int_{0}^{1} dx \ x^{i\nu} (1-x) f(1+i\eta, 2-i\eta; x)$$
(37)

with  $\eta = \eta_{12} + \eta_{13}$ ,  $f(\alpha, \beta; x) = (\varepsilon + x)^{-\alpha} (1 + \varepsilon x)^{-\beta}$ and  $\varepsilon = \frac{k_R - p_1}{k_R + p_1}$ ,  $\varepsilon$  being the small parameter in the vicinity of the resonance energy  $E_R$ . The investigation of the  $\varepsilon$ -dependence of the values  $A_j$ shows that only  $A_1$  has the resonant behavior

$$A_{1} = \frac{\Gamma(1+i\eta)B(1+i\nu, 2-i\nu)}{\epsilon^{1+i\xi}} {}_{2}F_{1}(1+i\nu; 1-i\eta; 1-\epsilon^{2}),$$
(38)

where the parameter  $\xi$  equals  $\eta - \nu$ .

As a result the resonant part of the reaction amplitude takes the form

$$T_{R}(\vec{k}_{23}\vec{p}_{1},\vec{p}_{0},E+i0) = \sum_{M} \frac{Y_{LM}(\vec{k}_{23})\chi_{23}(k_{23})}{E_{23} - \varepsilon_{R} + i\frac{\Gamma(E_{23})}{2}} e^{-\frac{\pi}{2}\xi} \Gamma(1+i\xi)\varepsilon^{-i\xi}\phi(\varepsilon)D_{M}\left(\frac{\vec{p}_{1}}{p_{1}}\right),$$
(39)

where

$$A_{1} = \frac{\Gamma(1+i\eta)B(1+i\nu, 2-i\nu)}{\varepsilon^{1+i\xi}} {}_{2}F_{1}(1+i\nu; 1-i\eta; 1-\varepsilon^{2})$$

$$(40)$$

and

$$D_M\left(\frac{\vec{p}_1}{p_1}\right) = e^{-\frac{\pi}{2}v} \Gamma\left(1+iv\right) \tilde{C}_M$$

At this stage there are a number of points to be made.

1. The expression (39) should be regarded as the leading term of the reaction amplitude asymptotic expansion in the parameter  $\varepsilon$ .

2. In the case of the resonance far from decay thresholds the parameterizations equivalent to (39) were obtained earlier in [1 - 4] by using different approaches: the eikonal approximation in [2]; the Redmond - Merkuriev approximation for the threebody Coulomb wave function in [4]; the approximate expression for the matrix elements of  $P_R^C$  in the momentum representation in [3]. Nevertheless, the condition  $|\varepsilon| << 1$  was pointed out explicitly only in [4].

3. The parameterizations [1 - 4] are valid if the supplementary condition  $|\varepsilon| << 1$  is fulfilled. In this case the function  $\phi(\varepsilon)$  (40) is practically equal to 1,

but the factor  $\phi(\varepsilon)$  has to be taken into account, if  $|\varepsilon\eta| \ge 1$ , for example, in reactions with heavy ions. The examples of the nuclear reactions, in which the condition under discussion is fulfilled (or is violated), are given in the Table.

4. The expression (39) describes the nearthreshold resonance formation as well. This case was originally investigated in [7, 8, 20] under condition  $|\varepsilon\eta| << 1$ . Redefining the values  $D_M$ , we transform the expression (39) to the results of [7, 8] (with the additional factor  $\phi(\varepsilon)$ )

$$T_{R}(\vec{k}_{23}\vec{p}_{1},\vec{p}_{0},E+i0) =$$

$$= \left[ e^{-\frac{\pi}{2}\xi} \Gamma\left(1+i\xi\right) \left(E_{23}-E_{R}+i\frac{\Gamma}{2}\right)^{-i\xi} \phi(\varepsilon) \right] \times$$

$$\times \frac{\chi_{23}(k_{23}) \sum_{M} Y_{LM}\left(\vec{k}_{23}\right) D_{M}}{E_{23}-\varepsilon_{R}+i\frac{\Gamma(E_{23})}{2}}.$$
(41)

The last term in Eq. (41) corresponds to the wellknown Migdal - Watson approximation [4, 10], whereas the factor in the first square bracket describes the influence of the accompanying particle Coulomb force field on the resonance decay.

Reactions and resonance		~						
decay channels	E <sub>p</sub> , MeV	E <sup>C</sup> , MeV	$E_R$ , MeV	Γ, MeV	ξ	$\eta=\eta_{12}+\eta_{13}$	ν	$ \varepsilon (E_R) $
<sup>7</sup> Li( $\alpha, \alpha$ ) <sup>7</sup> Li <sup>*</sup> , <sup>7</sup> Li <sup>*</sup> (7.45 MeV) $\rightarrow$ <sup>6</sup> Li+n	27.2	10.1	0.262	0.154	0 ÷ 0.04	0.45 ÷ 0.55	0.5	$2.0 \cdot 10^{-3}$
<sup>7</sup> Li( $\alpha, \alpha$ ) <sup>7</sup> Li <sup>*</sup> , <sup>7</sup> Li <sup>*</sup> (7.45 MeV) $\rightarrow \alpha$ +t	27.2	14.8	5.027	0.154	0 ÷ 0.15	$0.45 \div 0.65$	0.5	$0.2 \cdot 10^{-3}$
<sup>58</sup> Ni( <sup>6</sup> He, <sup>57</sup> Co) <sup>7</sup> Li <sup>*</sup> , <sup>7</sup> Li <sup>*</sup> (7.45 MeV)→ <sup>6</sup> Li+n	13.6	6.9	0.262	0.154	0 ÷ 5	34 ÷ 42	37.2	2.9 · 10 <sup>-3</sup>
<sup>7</sup> Li(d, $\alpha$ ) <sup>5</sup> He <sup>*</sup> , <sup>5</sup> He <sup>*</sup> (1.27 MeV) $\rightarrow \alpha + n$	2	16.68	2.059	5.57	0.23 ÷ 20	0.2 ÷ 20	0.25	0.5 · 10 <sup>-1</sup>
<sup>7</sup> Li(d, α) <sup>5</sup> He <sup>**</sup> , <sup>5</sup> He <sup>**</sup> (16.76 MeV)→α+n	6.8	20.4	17.67	0.09	0.1 ÷ 2.7	0.7 ÷ 3.3	0.6	4.1 · 10 <sup>-3</sup>
<sup>4</sup> He(d, p) <sup>5</sup> He <sub>g.s.</sub> , <sup>5</sup> He <sub>g.s.</sub> $\rightarrow \alpha + n$	11.3	2.1	0.89	0.6	0 ÷ 0.3	$0.2 \div 0.5$	0.3	6.3 · 10 <sup>-2</sup>
${}^{10}B(d, \alpha)^8Be^*,$ ${}^8Be^*(19.86 \text{ MeV}) \rightarrow \alpha + \alpha$	13.6	29.8	19.95	0.7	0.5 ÷ 3.2	0 ÷ 2.5	0.7	8.9 · 10 <sup>-3</sup>
${}^{58}$ Ni( <sup>7</sup> Li, ${}^{57}$ Co) <sup>8</sup> Be <sup>*</sup> , ${}^{8}$ Be <sup>*</sup> (19.86 MeV)→α+α	50	53.8	19.95	0.7	$0 \div 8$	6 ÷ 16	7.8	2.6 · 10 <sup>-3</sup>
<sup>58</sup> Ni( <sup>7</sup> Li, <sup>57</sup> Co) <sup>8</sup> Be <sup>*</sup> , <sup>8</sup> Be <sup>*</sup> (19.86 MeV)→α+α	200	187.6	19.95	0.7	0 ÷ 0.45	3.3 ÷ 3.9	3.5	5.2 · 10 <sup>-4</sup>

×

The parameters of the resonances, which are formed in the final states of different reactions

The parameterization (41) leads to the following expression for the value  $|T_R|^2$ :  $\left|T_R(\vec{k}_{23}\vec{p}_1, \vec{p}_0, E+i0)\right|^2 = e^{-\pi(\eta-\nu_1)} \left|\Gamma\left(1+i\eta-i\nu_1+i\nu_2\right)\right|^2 \times$ 

$$\frac{e^{2(\eta-\nu_{1})\operatorname{arcctg}\left(\frac{2}{\Gamma}(E_{R}-E_{23})\right)}|\chi_{23}(k_{23})|^{2}}{\left(\left(E_{23}-E_{R}\right)^{2}+\frac{\Gamma^{2}}{4}\right)^{-\nu_{2}}\left(\left(E_{23}-\varepsilon_{R}\right)^{2}+\frac{\Gamma^{2}(E_{23})}{4}\right)} \times$$

×

$$\sum_{M,M'} Y_{LM}(\vec{k}_{23}) Y^*_{LM'}(\vec{k}_{23}) D_M D^*_M.$$
(42)

The Coulomb parameter v is represented in (42) as the sum  $v = v_1 - iv_2$  with  $v_2 > 0$ . For resonances far from the decay thresholds the value  $v_2$  is negligibly small.

The parameterization predicts the following peculiarities:

1. At  $\eta - \nu_1 > 0$  the resonance position is shifted to the lower energies  $E_{23}$ . If resonance is far from the decay thresholds the resonance curve is broadened [1, 4], while in the case of near-threshold resonance the narrowing effect can be observed [7, 8].

2. If  $\eta - v_1 < 0$ , the position of the resonance is shifted in the direction of higher energies  $E_{23}$  and is always broadened [4, 7, 8].

3. In all cases the resonance curve is asymmetric.

4. If the parameter  $v_2$  is not small, the resonance curve is additionally broadened.

5. In case of the near-threshold resonances the decay branching ratio change is possible too [4, 11].

# The branching ratio for the decay of the nearthreshold resonance <sup>7</sup>Li<sup>\*</sup>(7.45 MeV)

In this section the properties of the near-threshold resonance  $^{7}\text{Li}^{*}(7.45\text{MeV})$  are investigated. This resonance is formed in the reaction

$$\alpha + {^{7}\text{Li}} \rightarrow \alpha + {^{7}\text{Li}} * \rightarrow \alpha + {^{6}\text{Li}} + n \qquad (43)$$
$$\rightarrow \alpha + \alpha + t$$

at  $E_{\alpha} = 27.2$  MeV. The scattered  $\alpha$ -particles were detected at  $\theta_{\alpha} = 44^{\circ}$ . We can use the amplitude parameterization (42) since the parameter  $\epsilon$  is small in this reaction ( $\epsilon_n \approx \epsilon_{\alpha} \approx 10^{-3}$ ).

For these kinematical conditions the resulting Coulomb parameters  $\xi = \eta - \nu$  for both reaction final channels are quite small

The double differential cross section is defined by the relation

$$\frac{d^2 \sigma_i}{dE_{\alpha} d\Omega_{\alpha}} = \frac{32\pi^4 n_1}{p_0} (E_{\alpha} E_{23})^{\frac{1}{2}} (m_{\alpha} \mu_{23})^{\frac{3}{2}} \int d\Omega_{23} \left| T(\vec{k}_{23} \vec{p}_1, \vec{p}_0, E + i0) \right|^2.$$
(46)

The integration over the direction of the momentum  $\vec{k}_{23}$  can be performed by the following way [7]. By introducing the spin variables

 $(J^{\pi} = \frac{5}{2}(P_{5/2}), L = 1, S = \frac{3}{2}$  - the spin of the resonant system) and summing over the final spins we can rewrite the expression (42) as

$$\left| T(\vec{k}_{23}\vec{p}_{1},\vec{p}_{0},E+i0) \right|^{2} = \frac{f(\xi) \cdot \left| \chi(k_{23}) \right|^{2}}{\left( \left( E_{23} - \varepsilon_{R} \right)^{2} + \frac{\Gamma^{2}(E_{23})}{4} \right)_{M,M'}^{\mu,m,m'}} \sum_{\substack{\mu,m,m'\\M,M'}} (LMS\mu \mid Jm)(LM'S\mu \mid Jm') \times D_{M}D_{M}^{*}Y_{LM}\left(\vec{k}_{23}\right)Y_{LM'}^{*}\left(\vec{k}_{23}\right),$$
(47)

 $(|\xi_n| \sim 0.04, |\xi_{\alpha}| \sim 0.15)$ , so the change of the resonance parameters is not pronounced (Fig. 1).

The more pronounced effect was observed under investigation of the decay branching ratio. The probability of the specified decay channel was defined by the relation [11, 17]

$$P_i = \frac{\sigma_i}{\sigma_1 + \sigma_2} \,. \tag{44}$$

In Eq. (44)  $\sigma_i$  denotes the result of the integration of the double differential cross section over the energy range, which corresponds to the resonant peak:



Fig. 1. The shape of the resonance <sup>7</sup>Li<sup>\*</sup>(7.45 MeV), decaying into the channel  $\alpha$  + t in the reaction  $\alpha$  + <sup>7</sup>Li $\rightarrow \alpha$  + <sup>7</sup>Li $\rightarrow \alpha$  +  $\alpha$  + t at E<sub> $\alpha$ </sub> = 27.2 MeV. The calculations on the base of the Migdal - Watson model and the parameterization (42) are shown by dashed and solid line, respectively.

It should be pointed out that in case of the resonances far from the decay thresholds the relation (44) is equal to  $\Gamma_i/\Gamma_{tot}$  as in the case, when the accompanying particle does not influence on the resonance decay [4].

where

$$f(\xi) = \frac{2\pi\xi}{e^{2\pi\xi} - 1} \exp\left[2\xi \cdot \operatorname{arcctg}\left(\frac{2(E_R - E_{23})}{\Gamma}\right)\right].$$
(48)

The function  $f(\xi)$  depends on the angle  $\theta_{1,23}$ between the momenta  $\vec{k}_{23}$  and  $\vec{p}_1$ , so we have

$$f(\xi) = \sum_{l} \frac{2l+1}{2} f_{l} P_{l}(\cos(\theta_{1,23})) = 2\pi \sum_{l,\lambda} f_{l} Y_{l\lambda}(\vec{p}_{1}) Y_{l\lambda}^{*}(\vec{k}_{23})$$
(49)

Owing to the properties of the spherical harmonics only even values of *l* give the contribution in the integral (46). In the reaction under investigation (43) ( $E_{\alpha} = 27.2 \text{ MeV}$ ,  $\theta_{\alpha} = 44^{\circ}$ ) the value  $f_0$  dominates ( $f_l \approx 10^{-4} f_0$ ,  $l \ge 1$ ), therefore

$$\int d\Omega_{23} |T|^2 = \frac{f_0 |\chi(k_{23})|^2}{\left( \left( E_{23} - \varepsilon_R \right)^2 + \frac{\Gamma^2(E_{23})}{4} \right)} \sum_M |D_M|^2.$$
(50)

The last expression shows that the influence of the accompanying  $\alpha$ -particle Coulomb field is described by the unique function  $f_0$ .

The vertex functions  $\chi_i(k_{23})$  and the energydependent width  $\Gamma_n(E_{23})$  were chosen in accordance with the formal resonance theory [10, 15, 16], in particular,  $\chi(k_{23}) \sim k_{23}^L$  and  $\Gamma_n(k_{23}) \sim k_{23}^{2L+1}$  (L=1).



Fig. 2. The probability of decay of  ${}^{7}\text{Li}^{*}(7.45 \text{ MeV})$  into the channel n +  ${}^{6}\text{Li}$  in the reaction  ${}^{7}\text{Li}(\alpha, \alpha){}^{7}\text{Li}^{*}$  at  $\text{E}_{\alpha} = 27.2 \text{ MeV}$  and  $\theta_{\alpha} = 44^{\circ}$  as a function of the parameter N =  $\Delta E/\Gamma$ .

The probability of decay depends on the interval of integration width  $\Delta E$  (Fig. 2): to obtain the accuracy of 5 % this width should be about  $10 \cdot \Gamma$ . Usually the integration interval of experimental resonance peaks does not exceed 5  $\cdot \Gamma$ . Therefore, the

probability decay into the  $n + {}^{6}Li$  channel  $P(n + {}^{6}Li)_{theor} = 0.58 \pm 0.06$  can be used as the result of calculations for comparison with experimental data. The above error covers all possible values of this quantity calculated for the range of integration width up to  $\Delta E > 20 \cdot \Gamma$ . The calculated decay probability agrees well with the experimental value  $P(n + {}^{6}Li)_{exp} = 0.56 \pm 0.03$ , which was obtained in [17] for  ${}^{7}Li^{*}(7.45 \text{ MeV})$  resonance excited in the reaction under investigation. The measurements in [17] were performed at  $E_{\alpha} = 27.2 \text{ MeV}$  and  $\theta_{\alpha} = 44^{\circ}$  for all possible decay angles of  ${}^{6}Li (\theta_{6Li} \text{ and } \phi_{6Li})$  using the position-sensitive detector and the method proposed in [11].

At the same time, both the experimental and theoretical results differ noticeably from the relation  $\Gamma(n + {}^{6}\text{Li})/\Gamma_{tot} = 0.77$  and  $\sigma_{n}/\sigma_{tot} = 0.71$  [18], where  $\sigma_{n}$  and  $\sigma_{tot}$  are the  $n + {}^{6}\text{Li}$  elastic and total cross sections at the resonance energy. The calculation of the decay probability for the binary reaction by analogy with (45) gives P(n + {}^{6}\text{Li}) = 0.68 instead of the value 0.71.

The performed calculations also showed that the value  $P(n + {}^{6}Li)$  strongly depends on the incident  $\alpha$ -particle energy (Fig. 3). At high energies the influence of the accompanying  $\alpha$ -particle on the resonance decay becomes negligible, so that the probability  $P(n + {}^{6}Li)$  is the same as in the isolated decay case.



Fig. 3. The dependence of the probability  $P(n + {}^{6}Li)$ in the reaction  ${}^{7}Li(\alpha, \alpha){}^{7}Li^{*}$  at  $\theta_{\alpha} = 44^{\circ}$  on the energy of the incident  $\alpha$ -particle: the interval of the integration width  $\Delta E$  is equal to  $5 \cdot \Gamma$  (solid line) and  $10 \cdot \Gamma$  (dashed line),  $\bullet$  – experimental data from [17].

The dependence of the probability of the decay  $^{7}\text{Li}^{*}(7.45\text{MeV})$  into the channel n +  $^{6}\text{Li}$  at different detecting angles of the  $\alpha$ -particle was calculated too (Fig. 4). Unfortunately, there are no more data in addition to those obtained in [17], which could confirm predicted energy and angular dependences of the decay probability into different channels for  $^{7}\text{Li}^{*}(7.45 \text{ MeV})$  resonance.



Fig. 4. The angular dependence of the decay probability  $P(n + {}^{6}Li)$  in the reaction  ${}^{7}Li(\alpha, \alpha){}^{7}Li^{*}$  at  $E_{\alpha} = 27.2$  MeV:  $\Delta E$  is equal to  $10 \cdot \Gamma$ , • – experimental data from [17].

## Conclusions

Various properties of decay of two-fragment nuclear resonances that are formed in three particle reactions are predicted by modified theory that takes into account the Coulomb interaction in the exit

- Komarov V.V., Green A.M., Popova A.M., Shablov V.L. Coulomb and nuclear field effects on two-body resonances // Modern Phys. Lett. A. - 1987. - Vol. 2. -P. 81 - 88.
- Kuchiev M.Yu., Sheinerman S.A. Resonance processes with three charged particle in the final state // Zh. Eksp. Teor. Fiz. - 1986. - Vol. 90. - P. 1680 - 1689.
- Ashurov A.R., Zubarev D.A., Mukhamedzhanov A.M., Yarmukhamedov R. Reactions with three charged particles and a resonance in the intermediate state// Sov. J. Nucl. Phys. - 1991. - Vol. 53. - 97 - 106.
- Komarov V.V., Popova A.M., Karmanov F.I. et al. Scattering properties of two-fragment systems produced by many-particle reactions // Phys. of Elem. Part. and Atom. Nucl. - 1992. - Vol. 23, No. 4. -P. 1035 - 1087.
- Nemetz O.F., Popova A.M., Komarov V.V. et al. Change in the scattering properties of two-fragment nuclear systems in many-particle nuclear reactions // Izv. Akad. Nauk SSSR: ser. fiz. - 1990. - Vol. 54, No. 5. - P. 942 - 948.
- Pugatch V.M., Kozir Yu.E., Medvedev V.I. et at. Quantum characteristics of short lived states of <sup>8</sup>Be excited in binary and three-particle reactions // Izv. Akad. Nauk SSSR: ser. fiz. - 1985. -Vol. 49. - P. 905 -910.
- Fazio G., Giardina G., Karmanov F.I., Shablov V.L. Properties of the Resonance Scattering in Two-Fragment Systems Formed in Many-Particle Nuclear Reactions // Int. J. Mod. Phys. E. - 1996. - Vol. 5. -P. 175 - 190.
- 8. *Shablov V.L., Tyras I.A.* Modeling the dynamics of the final state interaction in nuclear reactions with charged particles // Izvestia visshikh uchebnikh zavedeniy.

channel of such reactions. Some of them have experimental confirmation, while others require further experimental studies, especially in the case of near-threshold resonances, for which the change of decay branching ratio is possible. So far this phenomenon was observed only for near-threshold resonance <sup>7</sup>Li<sup>\*</sup>(7.45 MeV) at the decay into n + <sup>6</sup>Li channel in three-particle reaction <sup>7</sup>Li( $\alpha$ ,  $\alpha^{6}$ Li)n.

The regularities of the non-isolated resonance decay established in this work can be applied both to the interpretation of the experimental data and to the recovery of the resonance parameters using the three and four particle reaction final state data. The parameterization (41) can be useful to plan new experiments and to predict new effects in nonisolated decay of unstable quantum systems.

It should be expected that the effect of the branching ratio change discovered in the nonisolated <sup>7</sup>Li<sup>\*</sup>(7.45 MeV) decay could take place in other reactions with the formation of light nuclei resonant states, for example, <sup>5</sup>He<sup>\*</sup>(16.75 MeV), <sup>5</sup>Li(16.6 MeV), <sup>8</sup>Be(22.2 MeV). The investigation of this problem is now in progress.

#### REFERENCES

Yadernaya energetika. - 2007. - Vol. 3, No. 2. - P. 127 - 131.

- Nemets O.F., Pavlenko Yu.N., Shablov V.L. et al. Angular correllations and decay branching ratio for excited state of <sup>7</sup>Li\*(7,45 MeV) in reactions <sup>7</sup>Li(α, α)<sup>7</sup>Li\* // Nucl. Phys. At. Energy. - 2007. - Vol. 1(19). - P. 36 - 44.
- Komarov V.V., Popova A.M., Shablov V.L. Dynamics of Few Quantum Particle Systems. – M.: Moscow University, 1996. - 334 p.
- 11. *Pavlenko Yu.N.* The method of branching ratio measurements for nuclear unbound states produced by three particle reactions // Problems of atomic science and technology. Ser. Nucl. Phys. Inv. 2005. Vol. 6 (45). P. 11 16.
- Shablov V.L., Bilyk V.A., Popov Yu.V. Cook's method in the problem of the many-body Coulomb wave operator convergence // Fundamental and applied mathematics. - 2002. Vol. 8. - P. 559 - 566.
- Peterkop R.K. Theory of Ionization of Atoms. Boulder: Colorado Associated University Press, 1977.
   - 263 p.
- Nordsieck A. Reduction of an Integral in the Theory of Bremsstrahlung // Phys. Rev. - 1954. - Vol. 93. -P. 785 - 787.
- 15. Wildermuth K., Tang Y.C. A United Theory of Nuclei. - M.: Mir, 1980. - 502 p. (In Russian).
- Nikitiu N. Phase-shift analysis in Physics of Nuclear Interactions. – M.: Mir, 1983. - 416 p.
- 17. Pavlenko Yu.N., Shablov V.L., Bondarenko O.S. et al. Space distributions and decay probability for excited state of <sup>7</sup>Li\*(7.45 MeV) in reaction <sup>7</sup>Li( $\alpha$ ,  $\alpha^{6}$ Li)n // Nucl. Phys. At. Energy. - 2007. - Vol. 20 - P. 65 - 74.
- 18. Smith A.B., Guenther P.T., Whalen J.F. Neutron total

and scattering cross sections of  $^6\mathrm{Li}$  in the low-MeV range // Nucl. Phys. A. - 1982. - Vol. 373, No. 2. - P. 305 - 325.

- 19. *Reed M., Simon B.* Methods of Modern Mathematical Physics IV: Analysis of Operators. - New York, San Francisco, London: Academic Press, 1978. - 404 p.
- 20. Pavlenko Yu.N., Dobrikov V.N., Doroshko N.L. et al. Decay properties of short lived resonances of light nuclei in many-particle nuclear reactions // Int. J. of Mod. Phys. E. - 2010. Vol. 19, Iss. 5-6. - P. 1220 -1226.
- 21. *Merkuriev S.P., Faddeev L.D.* Quantum Scattering Theory for Few Particle Systems. - M.: Nauka, 1985. -400 p.
- Shablov V.L., Bilyk V.A., Popov Yu.V. The momentum representation of the two-body Coloumb Green's function // J. Phys. IV (France). 1999. Vol. 9, Pr 6. P. 65 69.
- 23. Baz'A.I., Zel'dovich Ya.B., Perelomov A.M. Scattering, Reactions, Decays in Nonrelativistic Quantum Mechanics. - M.: Nauka, 1971. - 544 p.

### А. В. Михайлов<sup>1</sup>, Ю. М. Павленко<sup>2</sup>, В. Л. Шаблов<sup>1</sup>, А. В. Степанюк<sup>2</sup>, І. А. Тирас<sup>1</sup>

<sup>1</sup> Обнінський інститут атомної енергетики, Національний дослідницький ядерний університет МІФІ, Обнінськ, Росія <sup>2</sup> Інститут ядерних досліджень НАН України, Київ

## ЕФЕКТИ КУЛОНІВСЬКОЇ ВЗАЄМОДІЇ В БАГАТОЧАСТИНКОВИХ ЯДЕРНИХ РЕАКЦІЯХ З УТВОРЕННЯМ ДВОФРАГМЕНТНИХ РЕЗОНАНСІВ

Розроблено модифіковану теорію взаємодії в кінцевому стані, що враховує кулонівську взаємодію продуктів розпаду двофрагментних ядерних резонансів із супутнім продуктом реакції, включаючи випадок біляпорогових резонансів. Досліджено також зміну співвідношення гілок розпаду для біляпорогового резонансу  $^{7}$ Li<sup>\*</sup>(E<sub>x</sub> = 7,45 MeB), що збуджується в реакції  $^{7}$ Li( $\alpha$ ,  $\alpha$ ) $^{7}$ Li<sup>\*</sup> при E<sub> $\alpha$ </sub> = 27,2 MeB.

*Ключові слова*: тричастинкові ядерні реакції, ядерні резонанси, теорія резонансів, кулонівська взаємодія, біляпорогові резонанси, канали розпаду, співвідношення гілок розпаду.

### А. В. Михайлов<sup>1</sup>, Ю. Н. Павленко<sup>2</sup>, В. Л. Шаблов<sup>1</sup>, А. В. Степанюк<sup>2</sup>, И. А. Тырас<sup>1</sup>

<sup>1</sup> Обнинский институт атомной энергетики, Национальный исследовательский ядерный университет МИФИ, Обнинск, Россия <sup>2</sup> Институт ядерных исследований НАН Украины, Киев

## ЭФФЕКТЫ КУЛОНОВСКОГО ВЗАИМОДЕЙСТВИЯ В МНОГОЧАСТИЧНЫХ ЯДЕРНЫХ РЕАКЦИЯХ С ОБРАЗОВАНИЕМ ДВУХФРАГМЕНТНЫХ РЕЗОНАНСОВ

Разработана модифицированная теория взаимодействия в конечном состоянии, которая учитывает кулоновское взаимодействие продуктов распада двухфрагментных ядерных резонансов с сопутствующим продуктом реакции, включая случай околопороговых резонансов. Исследовано также изменение соотношения вервей распада для околопорогового резонанса <sup>7</sup>Li\*( $E_x = 7,45$  MэB), возбуждаемого в реакции <sup>7</sup>Li( $\alpha, \alpha$ )<sup>7</sup>Li\* при  $E_\alpha = 27,2$  MэB.

*Ключевые слова*: трехчастичные ядерные реакции, ядерные резонансы, теория резонансов, кулоновское взаимодействие, околопороговые резонансы, каналы распада, соотношение ветвей распада.

Надійшла 11.11.2014 Received 11.11.2014