# SEMICLASSICAL INERTIA FOR NUCLEAR COLLECTIVE ROTATION 

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#### Abstract

The collective rotation motion is described within the local approximation of the semiclassical Gutzwiller trajectory approach to the response function theory through the cranking model. It is shown that the smooth local part of the moment of inertia for the collective rotation of deformed nuclei around the axis, perpendicular to the symmetry axis of the infinitely deep axially-symmetric square-well potential, is the rigid-body quantity. The "classical rotation" with the rigid-body inertia moment was found in the spherical limit.


## 1. Introduction

Many interesting phenomena were discovered from experimental data on the collective rotations in nuclei $[1-5]$. The powerful and successful theoretical tools for their study are suggested as the cranking model $[2,3,6-9]$ and the generalized collective model of Bohr, Mottelson and Rainwater [9-11]. The most of calculations have been done for the collective rotations around the axis perpendicular to the symmetry axis of nucleus. We point out also another effect, considered in connection with the nuclear rotation as the alignment of the individual angular momenta of particles along the symmetry axis of nucleus. Average of these individual excitations can be referred to the so called "classical nuclear rotation" [12] or the magnetic susceptibility of spherical metallic clusters and circle quantum dots [13]. Such phenomena were studied on basic of the mean field approach within the shell correction method [14, 15] with help of the semiclassical Periodic Orbit Theory (POT) [16-19]. The POT was successfully applied for explanation of the nuclear shell structures in the deformed system, which lead to the double-humped fission barrier (the second potential well) [16, 19-21] as well as for understanding the angular momentum alignment in the yrast-line energies of spherical [12] and deformed [22] nuclei. Another application of the POT is related to the study of supershell effects in the magnetic susceptibility of metallic clusters and quantum dots [13].

In this paper we perform the semiclassical cranking model calculations of the inertia moment for the collective rotation of nuclei around the axis perpendicular to the symmetry axis applying the abovementioned ideas within the response function formalism [10, 23] like for the transport coefficients [24-26]. The Section 2 follows general points of this approach. Section 3 deals with the semiclassical derivations for the Green's functions by means of
the mean field approach through the infinitely deep square-well potential with the deformed axiallysymmetric shapes. The semiclassical results for the inertia moment were obtained in the local approximation, similar to the Thomas - Fermi approach [26-33], in Section 4. A short discussion remarks are presented in Section 5.

## 2. Inertia moment for the collective rotation

Within the cranking model, the collective nuclear rotation around the axis, $x$, perpendicular to the symmetry axis of the axially symmetric mean field potential, $z$, can formally considered by solving the eigen-value problem for the single-particle perturbed Hamiltonian, called usually as the Routhian,

$$
\begin{equation*}
H_{\omega}=H-\omega l_{x}, \tag{1}
\end{equation*}
$$

where $H$ is the unperturbed single-particle Hamiltonian for a mean field, $l_{x}$ the angular momentum projection to the axis $x, \omega$ the Lagrangian multiplier frequency defined by the constraint for quantum average,

$$
\begin{equation*}
\left\langle l_{x}\right\rangle_{\omega} \equiv \sum_{i} \int \mathrm{~d} \mathbf{r} \psi_{i}^{\omega *}(\mathbf{r}) l_{x} \psi_{i}^{\omega}(\mathbf{r})=I_{x}, \tag{2}
\end{equation*}
$$

with a given angular momentum projection $I_{x}$ of nucleus to the axis $x, \psi_{i}^{\omega}(\mathbf{r})$ are the eigenfuntions and $\varepsilon_{i}^{\omega}$ the eigenvalues of the Routhian $\hat{H}_{\omega}$ Eq. (1). Thus, we may formally consider the moment of the inertia, $\Theta_{x}$, as the response of the quantum average, $\delta\left\langle l_{x}\right\rangle_{\omega}$, see Eq. (2), to the external cranking field, $-\omega l_{\star}$, like a susceptibility, similar to the magnetic or isolated susceptibilities [13, 23, 26, 34],

$$
\begin{equation*}
\delta\left\langle l_{\chi}\right\rangle_{\omega}=\Theta_{\chi} \delta \omega, \tag{3}
\end{equation*}
$$

where

$$
\begin{gather*}
\Theta_{x}=\frac{4}{\pi} \int_{0}^{\infty} \mathrm{d} \varepsilon n(\varepsilon) \times \\
\times \int \mathrm{d} \mathbf{r}_{1} \int \mathrm{~d} \mathbf{r}_{2} l_{x 1} l_{x 2} \operatorname{Re} G\left(\mathbf{r}_{1}, \mathbf{r}_{2}, \varepsilon\right) \operatorname{Im} G\left(\mathbf{r}_{1}, \mathbf{r}_{2}, \varepsilon\right) \\
G\left(\mathbf{r}_{1}, \mathbf{r}_{2}, \varepsilon\right)=\sum_{i} \frac{\psi_{i}^{*}\left(\mathbf{r}_{1}\right) \psi_{i}\left(\mathbf{r}_{2}\right)}{\varepsilon-\varepsilon_{i}+i \Gamma} \tag{4}
\end{gather*}
$$

$\Gamma=+0$ for the undamped motion, $l_{x \nu} \equiv l_{x}\left(\mathbf{r}_{v}\right)$ is the angular momentum projection of particle at the point $\mathbf{r}_{v}(v=1,2), n(\varepsilon)$ the Fermi occupation numbers, $n(\varepsilon)=1 /\{1+\exp [(\varepsilon-\lambda) / T])\}, \quad \lambda$ the chemical potential and $T$ the temperature, $\lambda \approx \mathcal{E}_{F}=\hbar^{2} k_{F}^{2} / 2 m$, $\varepsilon_{F}$ and $k_{F}$ are the Fermi energy and momentum, respectively, $m$ is the nucleon mass, see [3, 10]. Factor 2 takes into account the spin degeneracy. Notice also, for convenience of the semiclassical derivations, we included the $\hbar^{2}$ appearing usually in the cranking-model formulas into the angular momentum squared factor, $l_{x}^{2}$. The eigenfunctions, $\psi_{i}$, and eigenvalues, $\varepsilon_{i}$ are defined by the Hamiltonian, $H_{\omega}$, at $\omega=0$ within the lowest order of perturbation expansion in a small parameter, $\hbar \omega / \varepsilon_{F} \ll 1$. Substituting the second equation in Eq. (4) into the first expression one obtains the wellknown cranking model result for the inertia moment, see Refs. [3, 7, 10],

$$
\begin{equation*}
\Theta_{x}=\sum_{i j} \frac{\left.\left(n_{i}-n_{j}\right)\left|\langle i| l_{x}\right| j\right\rangle\left.\right|^{2}}{\varepsilon_{j}-\varepsilon_{i}-i \Gamma} . \tag{5}
\end{equation*}
$$

Here $n_{i}=n\left(\varepsilon_{i}\right)$ are the Fermi occupation numbers for the single-particle state $\mid i>$ with the eigenfunction $\psi_{i}$ and eigenvalue $\varepsilon_{i}$ of the Routhian, $H_{\omega}$, at the lowest order of the perturbation expansion above mentioned. $\left.\left|\langle i| l_{x}\right| j\right\rangle \mid$ are the matrix elements of the angular-momentum projection operator $l_{x}$ taken for the transition between two such states $\mid i>$ and $\mid j>$ at the same lowest order of perturbation expansion, $\omega=0$. Notice, it is possible to generalize this formalism considering the excitations, denoted above by variation symbol $\delta$, from the single-particle state of the spectrum, $\varepsilon_{\omega}$, with a given $\omega$ by formal replace, $\psi_{i} \rightarrow \psi_{i}^{\omega}, \varepsilon_{i} \rightarrow \varepsilon_{i}^{\omega}$ with $\psi_{i}^{\omega}$ and $\varepsilon_{i}^{\omega}$ being the eigenfunctions and eigenergies of the Routhian
operator $H_{\omega}$ at finite $\omega$. Then, we may consider the small perturbation of this Routhian, $-\delta \omega l_{x}$, like in Eq. (1). Repeating this procedure step by step we may get the inertia moment, $\Theta_{x}(\omega)$, as susceptibility, $\partial\left\langle l_{x}\right\rangle_{\omega} / \partial \omega$, at each finite $\omega$. The Lagrangian multiplier $\omega$ in Eq. (4) or in the cranking formula (5), modified in this way by $\omega$ dependence, can be excluded from the consistency condition (2) and thus, one obtains the non-adiabatic angular-momentum dependence of the inertia moment. However, in the following, for simplicity, we shall neglect such non-adiabatic effects.

## 3. Semiclassical approach

The Green's function $G\left(\mathbf{r}_{1}, \mathbf{r}_{2}, \varepsilon\right)$ can be found with help of the semiclassical expansion derived by Gutzwiller $[16,17]$ from the quantum path-integral propagator,

$$
\begin{align*}
& G\left(\mathbf{r}_{1}, \mathbf{r}_{2}, \varepsilon\right)=\sum_{\alpha} G_{\alpha}\left(\mathbf{r}_{1}, \mathbf{r}_{2}, \varepsilon\right)= \\
& =-\frac{1}{2 \pi \hbar^{2}} \sum_{\alpha}\left|J_{\alpha}\left(\mathbf{p}_{1}, t_{\alpha}, \mathbf{r}_{2}, \varepsilon\right)\right|^{1 / 2} \times  \tag{6}\\
& \times \exp \left[\frac{i}{\hbar} S_{\alpha}\left(\mathbf{r}_{1}, \mathbf{r}_{2}, \varepsilon\right)-\frac{i \pi}{2} \mu_{\alpha}\right]
\end{align*}
$$

The index $\alpha$ covers all classical paths inside the potential well, which connect the two spatial points $\mathbf{r}_{1}$ and $\mathbf{r}_{2}$ for a given energy $\varepsilon$, and $S_{\alpha}$ is the classical action along such trajectory $\alpha$. The $\mu_{\alpha}$ denotes the phase related to the Maslov index through the number of all caustic and turning points of the path $\alpha$ [16]. The oscillation amplitude in Eq. (6) depends on the classical trajectory stability measured by the Jacobian, $J_{\alpha}\left(\mathbf{p}_{1}, t_{\alpha} ; \mathbf{r}_{2}, \varepsilon\right)$, for transformation from the initial momentum $\mathbf{p}_{1}$ and time $t_{\alpha}$ of the particle motion along the trajectory $\alpha$ to its final coordinate $\mathbf{r}_{2}$ and energy $\varepsilon$. For closed orbits $\alpha, \mathbf{r}_{2} \rightarrow \mathbf{r}_{1}=\mathbf{r}$, a continuous axial symmetry related to the existence of families of the periodic orbits crossing a given point $\mathbf{r}$ in the spherical potential well was taken into account [19] in the semiclassical calculations of the oscillation amplitudes in Eq. (6). For such a family, this amplitude of the Green's function term, $G_{\alpha}$, in expansion (6) over $\alpha$, see Eqs. (16), (31) of Ref. [19], is enhanced by factor proportional to $\hbar^{-1 / 2}$ with respect to that in the case of the isolated trajectories given in the last equation of Eq. (6).

Among all classical trajectories $\alpha$, we may single out $\alpha_{0}$ which connects directly $\mathbf{r}_{1}$ and $\mathbf{r}_{2}$ without reflections from the potential well edge. For the Green's function $G$, Eq. (6), one has then a separation, $G=G_{\alpha 0}+G_{\text {osc }}$, which leads to the corresponding splitting of the slightly averaged level density trace, $g_{\mathrm{TF}}(\varepsilon)+g_{\text {osc }}(\varepsilon)$, into a smooth part of the Thomas - Fermi model, $g_{\mathrm{TF}}(\varepsilon)$, and its shell structure correction, $g_{\text {osc }}(\varepsilon)$. We shall use more exact smooth density $g_{\mathrm{ETF}}(\varepsilon)$ of the extended Thomas Fermi model, with including the surface and curvature $\hbar$ corrections instead of the volume part $g_{\text {TF }}(\varepsilon)$, see Refs. [16-19]. The POT sum over the periodic orbits, $g_{\text {osc }}(\varepsilon)$, describes the shell effects in the single-particle spectrum. The corresponding split of the level density, $g_{\mathrm{TF}}(\varepsilon, \mu)+g_{\text {osc }}(\varepsilon, \mu)$, into a smooth Thomas - Fermi, $g_{\text {TF }}(\varepsilon, \mu)$, and oscillating periodic-orbit components $g_{\text {osc }}(\varepsilon, \mu)$ with the fixed angular momentum projection $\mu$ to the symmetry axis was suggested in [35] for the semiclassical study of the yrast-line properties, in particular, the shell effects in the inertia moment due to the alignment in spherical nuclei.

For calculations of the semiclassical moment of inertia, $\Theta_{x}$, one can substitute the trajectory expansion of Green's function (6) into Eq. (3) for $\Theta_{x}$. Here, we have to deal with both closed ( $\mathbf{r}_{1}=\mathbf{r}_{2}$ ) and non-closed $\left(\mathbf{r}_{1} \neq \mathbf{r}_{2}\right)$ trajectories $\alpha$, in contrast to the calculations of level density trace.

For the calculations of the moment of inertia, we shall consider a simple mean field in terms of the axially symmetric square-well potential with infinite walls like spheroidal cavity, and need to study separately the two different cases like in [26]:
(i) the nearly local case, $S_{\alpha}\left(\mathbf{r}_{1}, \mathbf{r}_{2}, \varepsilon_{F}\right) / \hbar=k_{F} L_{\alpha} \lesssim 1$,
(ii) non-local contributions, $k_{F} L_{\alpha} \gg 1$,
where $L_{\alpha}$ is the length of the trajectory $\alpha$, for instance, in spheroidal cavity, $k_{F}$ the Fermi momentum in units of $\hbar, k_{F}=\sqrt{2 m \varepsilon_{F}} / \hbar$.

In the case (i), after the Strutinsky averaging [14 - 16], the most important contribution is coming from the trajectory, $\alpha=\alpha^{\prime}=\alpha_{0}$, with a short length, $L_{\alpha 0}=s=\left|\mathbf{r}_{2}-\mathbf{r}_{1}\right| \lesssim 1 / k_{F} \ll R$, for large semiclassical parameter, $k_{F} R \gg 1$, where $R$ is the size of the nucleus. For simplicity of calculations of the contributions (i), $\alpha=\alpha^{\prime}=\alpha_{0}$, into Eq. (3) for the moment of inertia, $\Theta_{x}$, the variables $\left\{\mathbf{r}_{1}, \mathbf{r}_{2}\right\}$ can be
transformed to $\{\overline{\mathbf{r}}, \mathbf{s}\}, \quad \overline{\mathbf{r}}=\left(\mathbf{r}_{1}+\mathbf{r}_{2}\right) / 2, \mathbf{s}=\mathbf{r}_{2}-\mathbf{r}_{1}$. The Green's function $G_{\alpha 0}$ from Eq. (6) in the new variables $\overline{\mathbf{r}}, \mathbf{s}$ for small enough length $s$ of the trajectory $\alpha_{0}, s / R \ll 1$, is reduced approximately to a simple analytical form $G_{0}$ for free particle motion

$$
\begin{align*}
G_{\alpha 0}\left(\mathbf{r}_{1}, \mathbf{r}_{2}, \varepsilon\right) & \approx G_{0}\left(\mathbf{r}_{1}, \mathbf{r}_{2}, \varepsilon\right)=-\frac{m}{2 \pi \hbar^{2} s} \exp (i k s) \\
s & =\left|\mathbf{r}_{1}-\mathbf{r}_{2}\right|, \quad k=\sqrt{\frac{2 m \varepsilon}{\hbar^{2}}} \tag{7}
\end{align*}
$$

We are going to use this expression for the Green's function in the local approximation (i) in order to derive the smooth moment of inertia Eq. (4).

## 4. The local moment of inertia

After the transformation of the integrals over $\mathbf{r}_{1}$ and $\mathbf{r}_{2}$ in the term $\alpha=\alpha^{\prime}=\alpha_{0}$ of Eq. (4) to the new variables $\overline{\mathbf{r}}$ and $\mathbf{s}$ one can calculate analytically the inertia moment, $\Theta_{x}$, within the local (i) approximation (7), $G_{\alpha 0}\left(\mathbf{r}_{1}, \mathbf{r}_{2}, \varepsilon\right) \approx G_{0}\left(\mathbf{r}_{1}, \mathbf{r}_{2}, \varepsilon\right) \equiv G_{0}(s, \varepsilon)$, $l_{x 1} l_{x 2} \approx l_{x}^{2}$,

$$
\begin{align*}
\Theta_{x}= & \frac{4}{\pi} \int_{0}^{\infty} \mathrm{d} \varepsilon n(\varepsilon) \int \mathrm{d} \overline{\mathbf{r}} \int \mathrm{~d} \mathbf{s} l_{x}^{2} \operatorname{Im} G_{0}(s, \varepsilon) \operatorname{Re} G_{0}(s, \varepsilon)= \\
& =\frac{m^{2}}{2 \pi^{3} \hbar^{4}} \int_{0}^{\infty} \mathrm{d} \varepsilon n(\varepsilon) \int \mathrm{d} \overline{\mathbf{r}} \int \mathrm{~d} l_{x}^{2} \frac{\sin (2 k s)}{s^{2}} \tag{8}
\end{align*}
$$

For simplicity, as mentioned above, we neglected the non-adiabatic effects related to the $\omega$ (or angular moment, according to the constraint (2)) dependence of the inertia moment.

As shown in Figure, for the integration over $\mathbf{s}$ in the second line of Eq. (8) it is convenient to use the local $\left(\mathbf{r}_{2} \rightarrow \mathbf{r}_{1}\right)$ spherical coordinate system with the centre at the point $\mathbf{r}_{1} \approx \overline{\mathbf{r}}=\mathbf{r}$ and the polar axis $z_{s}$ crossing the potential symmetry centre and the point $\mathbf{r}$, Figure. For this integration we use now the explicit dependence of the classical single-particle angular momentum projection $l_{x v}(v=1,2)$ on the spatial coordinates, $\mathbf{r}_{v}$, and momentum, $\mathbf{p}_{v}$, i.e. the projection of the classical angular momentum $\mathbf{l}_{v}=\mathbf{r}_{v} \times \mathbf{p}_{v}$ on the axis $x$. The explicit expression for the angular momentum projection operator, $l_{x}$, as applied specifically for the rotation of the axiallysymmetric potential well around the axis, $x$, perpendicular to its symmetry axis, $z$, is given by

$$
\begin{equation*}
l_{x}^{2}=r_{\perp}^{2} p^{2}, \quad r_{\perp}^{2}=y^{2}+z^{2} \tag{9}
\end{equation*}
$$



A short trajectory $\alpha_{0}$ and the spherical coordinate systems with polar axis's $z$ and $z_{s}$ used for the integrations. The circle orbit of particle in the plane $y, z$ for the rotation around the axis x perpendicular to the symmetry axis $z$ is shown by dashed line; $\mathbf{p}$ is the particle momentum tangent to this orbit in the same plane.

Therefore, the integrals over $\mathrm{d} \mathbf{s}=s^{2} \mathrm{~d} s \sin \theta_{s} \mathrm{~d} \theta_{s} \mathrm{~d} \varphi_{s} \quad$ can be taken analytically within the limits in $s$ from 0 to $2 R(\theta)(R(\theta)$ is the surface radius for a given angle $\theta$ of the radius vector $\mathbf{r}$ ), and in all angles, $\theta_{s}$ from 0 to $\pi$, and $\varphi_{s}$ from 0 to $2 \pi$.

Applying the Strutinsky smoothing in spectrum [14-16, 19] to Eq. (8) with Eq. (9) for the angular momentum projection $l_{x}^{2}$

$$
\begin{align*}
& \tilde{\Theta}_{x}=\frac{m^{2}}{2 \pi^{3} \hbar^{4}} \int_{0}^{\infty} \mathrm{d} \varepsilon \tilde{n}(\varepsilon) p^{2} \times \\
& \times \int \mathrm{d} \overline{\mathbf{r}}\left(y^{2}+z^{2}\right)\left\langle\int \mathrm{d} \mathbf{s} \frac{\sin (2 k s)}{s^{2}}\right\rangle_{a v} \tag{10}
\end{align*}
$$

and calculating the integral over $\mathbf{s}$ in the spherical coordinate system shown in Figure, we note that the average of $\sin ^{2}(2 k R(\theta))$ in spectrum, $k$, marked by $\langle\ldots\rangle_{a v}$ is $1 / 2$. Here, $\tilde{n}$ is the smooth Strutinsky average of the occupation numbers [14]. By making use also the expression for the averaged particle density, equal approximately to the extended Thomas - Fermi particle density,

$$
\begin{equation*}
\tilde{\rho}=2 \int \frac{\mathrm{~d} \mathbf{p}}{(2 \pi \hbar)^{3}} \tilde{n}(\varepsilon), \tag{11}
\end{equation*}
$$

one finally arrives at the moment of inertia,

$$
\begin{equation*}
\tilde{\Theta}_{x}=m \tilde{\rho} \int_{V} \mathrm{~d} \mathbf{r}\left(y^{2}+z^{2}\right) \tag{12}
\end{equation*}
$$

where the integral is taken over the volume of the axially-symmetric cavity, V. Substituting the Fermi occupation numbers $\tilde{n}(\varepsilon)$ into the particle density, $\tilde{\rho}$ Eq. (11), we may find the temperature dependence of the inertia moment, $\tilde{\Theta}_{x}$ Eq. (12), for small enough temperature, $T \ll \varepsilon_{F}$,

$$
\begin{gather*}
\tilde{\rho}=\frac{1}{\pi^{2} \hbar^{3}} \int_{0}^{\infty} p^{2} \mathrm{dp} \tilde{\mathrm{n}} \approx \rho_{\mathrm{TF}}\left[1+\frac{\pi^{2}}{8}\left(\frac{\mathrm{~T}}{\varepsilon_{F}}\right)^{2}\right], \\
\rho_{\mathrm{TF}}=\frac{\mathrm{p}_{F}^{3}}{3 \pi^{2} \hbar^{3}}, \tag{13}
\end{gather*}
$$

where $\rho_{\mathrm{TF}}$ is the main (volume) term of particle density of the extended Thomas - Fermi model. Here, we integrated over modulus $p$ of the particle momentum $\mathbf{p}$ in the spherical coordinate system in Eq. (11), $\varepsilon=p^{2} / 2 m$, by parts and transformed the integration variable $p$ to a new variable $\xi=(\varepsilon-\lambda) / T$. As usually, extending approximately the low $\xi$-integration limit to $-\infty$ for $T \ll \lambda \approx \varepsilon_{F}$ in Eq. (13), we used the Sommerfeld expansion of the smooth relatively multiplier in front of the sharp bell-like derivative, $\mathrm{d} \tilde{\mathrm{n}}(\xi) / \mathrm{d} \xi$, in small parameter $T / \varepsilon_{F}$ at the second order.

As expected, the expression (12) is general for the rigid-body inertia moment [36] valid for a statistically equilibrium rotation of any cavities with axially-symmetric shapes of the potential surface [37]. This general result is also in agreement with obtained in [8] by direct applying numerically the Strutinsky averaging procedure $[14,15]$ to the cranking-model expression (5). In particular, for the spheroidal cavity with axis's, $a=b$, and $c$ for its arbitrarily deformed surface, one gets well-known expression [36],

$$
\begin{gather*}
\tilde{\Theta}_{x}=\frac{4 \pi m}{15} a^{2} c \tilde{\rho}\left(a^{2}+c^{2}\right)=\frac{1}{5} m A\left(a^{2}+c^{2}\right) \\
\tilde{\rho}=\frac{A}{V}, \quad V=\frac{4 \pi}{3} a^{2} c \tag{14}
\end{gather*}
$$

where $V$ is the volume of spheroid. For the case of the spherical limit of the infinitely deep square-well potential one naturally obtains the standard rigidbody expression too [36],

$$
\begin{equation*}
\tilde{\Theta}=\frac{8 \pi}{15} m \tilde{\rho} R^{5}=\frac{2}{5} m A R^{2} \tag{15}
\end{equation*}
$$

where $R$ is the radius of the spherical cavity, equivalent to the spheroid, $\tilde{\Theta}_{x}=\tilde{\Theta}_{y}=\tilde{\Theta}_{z}=\tilde{\Theta}$. For simplicity, we neglect small temperature corrections at the final results in (14), (15). This rigid-body result for the inertia moment is also in agreement with the semiclassical derivations for "classical rotation" around the symmetry axis of spherical nuclei in [12].

## 5. Discussion

We obtained the rigid-body inertia moment (12) for the collective rotation of the axially-symmetric nucleus in the mean-field approximation with the corresponding shapes of the infinitely deep squarewell potentials around the axis perpendicular to the symmetry axis, as expected for the statistically equilibrium rotation of the Fermi gas incorporated into the axially symmetric cavity [37]. This is in agreement with many other results for the statistically averaged inertia moment obtained within the cranking model [2, 8, 12], as a good test for our more general approach. The result (12) was derived in general case for any deformations of axially-symmetric shapes of the potential surface, in particular for the spheroid cavity with arbitrary deformation, $c / a$, see Eq. (14). It is interesting that in the spherical limit one has the finite nonzero value (15), in contrast to pure quantum
approaches [2, 8]. Following Ref. [12], we may consider that as the "classical rotation" which appears after the statistical averaging in spectrum like the Strutinsky averaging. Similarly, we may find "classical rotation" of the axially-symmetric shapes of the potential well around the symmetry axis, $\Theta_{z}=m \tilde{\rho} \int_{V} \mathrm{~d} \mathbf{r}\left(x^{2}+y^{2}\right)$, as averaging over many individual angular momenta of particles in their alignment along the symmetry axis.

The non-local contributions (ii) into the moment of the inertia $\Theta_{x}$ (3) can be obtained by substituting the other oscillating components of Green's function, $G_{\text {osc }}$, of the Gutzwiller trajectory expansion (6) related to the shell effects. In further publications we are going to show that the shell component of the moment of inertia can be obtained in the analytical form through the periodic orbits by calculating the integrals in Eq. (3) by the stationary phase method.

We thank very much Profs. V. Pashkevich, S. Frauendorf, H. Hofmann, F. Ivanyuk, V. M. Kolomietz and K. Matsuyanagi for many helpful discussions.

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# КВАЗІКЛАСИЧНИЙ МОМЕНТ ІНЕРЦІЇ ДЛЯ ЯДЕРНИХ КОЛЕКТИВНИХ ОБЕРТАНЬ 

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Колективний обертальний рух описується в локальному наближенні квазікласичного траєкторного підходу Гуцвіллера до теорії функцій відгуку за допомогою кренкінг-моделі. Отримано гладку локальну частину моменту інерції колективних обертань деформованих ядер навколо осі, перпендикулярної до осі симетрії нескінченно глибокої аксіально-симетричної потенціальної ями, у вигляді моменту інерції твердого тіла. Показано існування "класичного обертання" при наближенні до сферичної форми ядра.

## КВАЗИКЛАССИЧЕСКИЙ МОМЕНТ ИНЕРЦИИ ДЛЯ ЯДЕРНЫХ КОЛЛЕКТИВНЫХ ВРАЩЕНИЙ

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Коллективное вращательное движение описывается в локальном приближении квазиклассического траекторного подхода Гуцвиллера к теории функции отклика с помощью кренкинг-модели. Получен гладкий компонент момента инерции коллективного вращения деформированных ядер вокруг оси, перпендикулярной к оси симметрии бесконечно глубокой аксиально-симметричной потенциальной ямы, в виде момента инерции твердого тела. Показано существование "классического вращения" в пределе к сферической форме ядра.

