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THE COULOMB SUM OF ^{7}Li

The experimental values of longitudinal response function of the ^{7}Li nucleus have been obtained and these results have been used as the basis to find the Coulomb sum values at momentum transfers ranging from 0.55 to 1.625 fm^{-1}. The obtained experimental Coulomb sum values have been used to determine the total Coulomb energy of the ^{7}Li nucleus. The result of the comparison of the Coulomb energy for the ^{7}Li nucleus with the Coulomb energies for the ^{6}Li and ^{4}He nuclei a) is consistent with the paradox (revealed in the elastic electron scattering experiment) that the ^{7}Li rms charge radius is smaller than the one of the ^{6}Li nucleus; b) leads to the conclusion that, within the framework of the two-cluster model of the ^{7}Li nucleus (α + t), the size of the α-cluster should be larger than the one of the ^{4}He nucleus.

Keywords: electron scattering, ^{7}Li, longitudinal response function, Coulomb sum, Coulomb energy, clusters.

1. Introduction

The Coulomb energy is the part of nuclear energy that arises as a result of electrostatic (Coulomb) interaction between intranuclear protons. This interaction is one or two orders of magnitude weaker than the nuclear interaction, and yet, its role in the nuclear structure and nuclear reactions is considerable. For example, it is just the Coulomb interaction that determines the maximum size of atomic nuclei, or, in other words, the maximum number of protons that can be present in the stable nucleus.

The Coulomb energy has been investigated in a good many experiments. Based on the hypothesis of the isotopic invariance of nuclear forces, the investigators measured the Coulomb energy differences of mirror nuclei (e.g., see survey [1]). However, the experimental total Coulomb energy values ($E_{\rm coul}$) were obtained only for the nuclei ^{6}Li [2, 3], ^{7}Li [3], ^{12}C [4] and ^{4}He [5]. A small number of these values is explained by the fact that for determining $E_{\rm coul}$ it is necessary to know the Coulomb sum values ($S_{\rm L}(q)$) of the nucleus under study at 3-momentum transfers q = 0.8 ÷ 1.8 fm^{-1}. However, the determination of $S_{\rm L}(q)$ values appears to be a complicated and time consuming problem. So far, the experimental $S_{\rm L}(q)$ values at $q < 2$ fm^{-1} have been obtained for 10 nuclei apart from lithium isotopes. These are the results of work performed at laboratories of Saclay, Bates and KIPT. The Saclay and Bates teams carried out the measurements mainly at $q \geq 1.5$ fm^{-1}, while the Kharkiv team – at $q \leq 1.5$ fm^{-1}. [1]

[1] The given references refer to the works carried out after 1976. The earlier data of the works on the Coulomb sums, not mentioned here, had a relatively low accuracy, though they much contributed to gaining the experience for subsequent measurements.

Previously, we found the $S_{\rm L}(q)$ values for lithium isotopes at q = 0.750 ÷ 1.625 fm^{-1} [6 - 8]. In the present work, we have extended the range of momentum transfers by estimated $S_{\rm L}(q)$ values for ^{7}Li to q = 0.55 and 0.65 fm^{-1}. As it will be shown below, the obtained array of experimental $S_{\rm L}(q)$ values has permitted us to determine the Coulomb energy of ^{7}Li to a higher accuracy than that given in [2 - 5]. However, the most accurate determination of the ^{7}Li Coulomb energy by itself is not the ultimate goal of the present work.

The nuclei of lithium isotopes are strongly clusterized. The difference between the types of their clusterization may probably be the reason why the charge radius of the ^{7}Li nucleus is smaller than the one of the ^{6}Li nucleus. This paradox was first revealed in [9], where the ratio of rms charge radii of lithium isotopes was found to be $\langle r^2\rangle^{1/2}(^7\mathrm{Li})/\langle r^2\rangle^{1/2}(^6\mathrm{Li}) = 0.948 \pm 0.008$. Considering that the Coulomb energy of the nucleus is dependent on its clusterization (see [2]), it would be of interest to compare the Coulomb energies of the ^{7}Li, the ^{6}Li and the ^{4}He nuclei.

2. Equations and formulae

The experimental determination of the Coulomb energy of nucleus is based on the equation for $E_{\rm coul}$ taken from [10], which relates $E_{\rm coul}$ to the data measured in the experiments on electron scattering by nuclei, viz., the squared charge form factor of the nuclear ground state $F_{\rm el}^2(q)$ (hereinafter referred to as nuclear form factor) and the Coulomb sum of the nucleus $S_{\rm L}(q)$. It should be noted that this equation is model-free, i.e., it is independent of any assumptions about the structure of the nucleus under study. The $E_{\rm coul}$ equation can be written as

$$E_{\text{coul}} = \frac{e^2}{\pi}\left(I_1 - I_2\right),$$

$$I_1 = \int_0^{\infty} Z^2 F_{\text{el}}^2(q)\left(1+\frac{q^2}{4M^2}\right)dq,$$

$$I_2 = \int_0^{\infty} Z\, G_{\text{E,p}}^2\left(q^2\right)\left(1 - S_{\text{L}}(q)\right)dq, \qquad (1)$$

where e – the elementary charge; M and $G_{\text{E,p}}(q^2)$ – the mass and electrical form factor of the proton, respectively; Z – the charge number of the nucleus.

The squared charge form factor of the nuclear ground state is defined as

$$F_{\text{el}}^2(q) = \frac{1}{Z^2\sigma_{\text{M}}(\theta, E_0)}\frac{d\sigma}{d\Omega}(\theta, E_0)\,\zeta, \qquad (2)$$

where $\frac{d\sigma}{d\Omega}(\theta, E_0)$ and $\sigma_{\text{M}}(\theta, E_0) = \left(\frac{e^2}{2E_0}\right)^2 \cos^2\frac{\theta}{2}\Big/\sin^4\frac{\theta}{2}$ are, respectively, the elastic electron-nucleus scattering cross-section and the Mott cross-section, i.e., the cross-section for electron scattering by the point spinless unit charge with infinite mass; E_0 – the initial energy of electron scattered through the angle θ; $\zeta = 1 + 2\frac{E_0}{AM}\sin^2\frac{\theta}{2}$ – the kinematic correction; A – the atomic mass of the nucleus.

In this paper, the symbol q denotes the effective 3-momentum transfer

$$q = \left\{4E_{\text{eff}}\left(E_{\text{eff}} - \omega\right)\sin^2(\theta/2) + \omega^2\right\}^{1/2}, \qquad (3)$$

where $E_{\text{eff}} = E_0 + 1.33\,Ze^2/\langle r^2\rangle^{1/2}$ – the effective energy; ω – the energy transfer to the nucleus. In the expression for E_{eff} the second term takes into account the nuclear electrostatic field effect on the incident electron [11].

The Coulomb sum has the form

$$S_L(q) = \int_{\omega_{el}^{+}}^{\infty} \frac{R_{\text{L}}(q,\omega)}{\tilde{G}_{\text{L}}^2\left(q_\mu^2\right)}\, d\omega, \qquad (4)$$

where the lower limit of the integral ω_{el}^{+} shows that the range of integration begins from the peak of elastic electron scattering by the nucleus, but the peak itself does not enter into the integral; the denominator of the integrand (4) is given by

$$\tilde{G}_{\text{L}}^2\left(q_\mu^2\right) = \frac{1 + q_\mu^2/4M^2}{1 + q_\mu^2/2M^2}\left[ZG_{\text{E,p}}^2\left(q_\mu^2\right) + NG_{\text{E,n}}^2\left(q_\mu^2\right)\right],$$

where q_μ – the 4-momentum transfer to the nucleus, its square being written as $q_\mu^2 = q^2 - \omega^2$; N – the neutron quantity in the nucleus; $G_{\text{E,n}}(q_\mu^2)$ – the electrical neutron form factor; $R_{\text{L}}(q, \omega)$ – the longitudinal response function, which together with the transverse response function $R_{\text{T}}(q, \omega)$ represents the expansion of the double-differential cross-section for electron-nucleus scattering $d^2\sigma(\theta, E_0, \omega)/d\Omega d\omega$ by the known expression [12], which can be written as

$$\frac{d^2\sigma(\theta, E_0, \omega)}{d\Omega d\omega} = \sigma_{\text{M}}(\theta, E_0)\left[\frac{q_\mu^4}{q^4}R_{\text{L}}(q,\omega) + \left(\frac{1}{2}\frac{q_\mu^2}{q^2} + \tan^2\frac{\theta}{2}\right)R_{\text{T}}(q,\omega)\right]. \qquad (5)$$

The given set of formulas demonstrates the relationship between the quantities used in Eq. (1) and the cross-sections measured in the electron-nucleus scattering experiments.

3. Experimental Coulomb sum values of the ^{7}Li nucleus

It follows from Eq. (1) that the Coulomb energy of the nucleus is the function of three physical quantities, and its value can be found from their experimental values. These quantities are: i) the nuclear form factor $F_{\text{el}}(q)$ of ^{7}Li (was measured in [9, 13]); ii) the electrical proton form factor $G_{\text{E,p}}(q_\mu^2)$ (the recent measured data can be found in [14]); iii) the Coulomb sum $S_{\text{L}}(q)$ of ^{7}Li (was measured in [6] at $q = 1.250 \div 1.625$ fm^{-1}, and in [8] at $q = 0.750 \div 1.125$ fm^{-1}). It should be noted that the accuracy of E_{coul} calculation is to a large extent dependent on the precision and range of experimental $S_{\text{L}}(q)$ values. Therefore, it was carried out a thorough re-processing of all our experimental data on ^{7}Li nucleus. For the purpose, we used improved data processing techniques developed by us in recent years. As a result, experimental $S_{\text{L}}(q)$ values have been obtained in a wider range of momentum transfers, and these quantities have been determined with higher accuracy than earlier ones.

The ^{7}Li measurements were performed using the spectrometer SP-95 of the KIPT electron linear accelerator LUE-300. The experimental facility and the measurement technique have been described in detail in a number of publications (e.g., see [7, 15, 16]). Therefore, we mention here only the ^{7}Li spectrum measurement conditions.

As it follows from Eq. (5), the determination of response functions from the spectra of scattered electrons calls for the measurements at different scattering angles and initial electron energies. Thus, the measurements on ^{7}Li were carried out in the following ranges: $\theta = 34.2 \div 160°$, $E_0 = 104 \div 259$ MeV. A total of 21 spectra were measured on ^{7}Li nuclei; and 28 spectra - on ^{12}C nuclei. The last spectra were necessary for normalization of the measured data for ^{7}Li. The processing of the data has given the longitudinal response function values of ^{7}Li (Fig. 1) at eight 3-momentum transfers: q = 0.750, 0.875, 1.000, 1.125, 1.250, 1.375, 1.500, 1.625 fm^{-1}. Substituting these experimental $R_L(q, \omega)$ into Eq. (4), we find the Coulomb sum values.

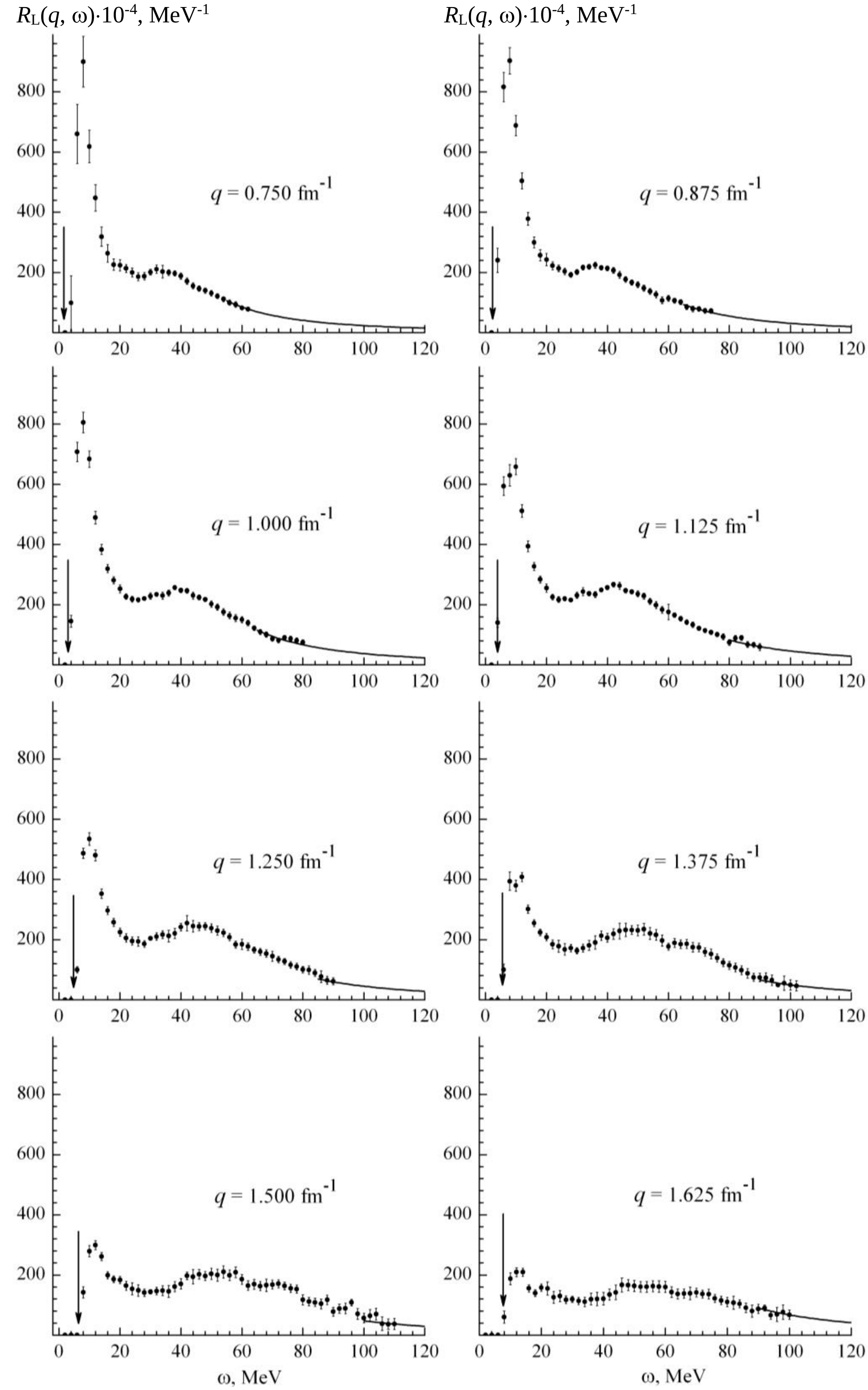

Fig. 1. Experimental longitudinal response function of ^{7}Li.
The vertical arrows indicate the peak position of elastic electron scattering by the nucleus.

4. Experimental data and processing techniques

We shall mention briefly the data processing procedures, which we have developed or modified in the recent years. These methodical developments were applied in the last revision of the measured data processing and, partially, in the work [8].

- The background from the (e^+e^-)-pair photoproduction by the target has been considered. The computation program for the effect has been written and tested. The computation by the program has demonstrated the negligibly small contribution of the process to our measurement results [17].

- A new computation program, which makes use of the whole cumbersome mathematical apparatus of [18, 19], has been written for radiative correction (rad. correction) of experimental spectra. That makes the rad. correction computations as accurate as those performed in the best foreign laboratories. Besides, we have analyzed the possibility of using the "equivalent radiator" approximation for calculating the radiation tail of the elastic scattering peak [20]. The application of the new rad. correction program to our data has left practically unchanged the previous $S_L(q)$ values measured at $q \geq 1$ fm^{-1}, whereas at lower momentum transfers the variations in $S_L(q)$ did not exceed the half of the experimental error.

- All investigations, where the response functions are derived from the experimental cross-sections, by all means include the data interpolation (e.g., see [7, 21]). We made an attempt to estimate the uncertainty, which may be introduced by this procedure to the calculated $S_L(q)$ values. For this purpose, while processing the data in the present work, we have used four different variants of interpolation, and have considered the caused-by-the-technique spread in the $S_L(q)$ values at different momentum transfers. As a result, it was found that the interpolation-induced uncertainty could be estimated to be 0.7 % of the $S_L(q)$ values.

- The expression for the Coulomb sum includes the squared electrical proton form factor $G_{E,p}^2(q^2)$. For the purpose of its calculation, the dipole formula and the estimation from [23] were used in [6] and in [7, 8], respectively. In the present work, we have used the $G_{E,p}^2(q^2)$ values taken from [14] being the last work on the subject. In the range of $q = 0.5 \div$ $\div\ 1.6$ fm^{-1}, the difference between the $G_{E,p}^2(q^2)$ values by the dipole formula and from [14] and [23] reaches several percent. In [14], Bernauer et al. have indicated the error corridor for the $G_{E,p}^2(q^2)$ values, which shows that at the considered momentum transfers the value of $\delta G_{E,p}^2(q^2)$ varies from 0.2 to 0.6 %.

- The excitation energy of the first level of ^{7}Li amounts to 0.47 MeV. In the measured spectra this level was not separated from the elastic scattering peak, and in all our previous publications its contribution to $S_L(q)$ was neglected. In the present work, the contribution from this level (about 2 %) was taken from the measurements of [9], and was included in the $S_L(q)$ values.

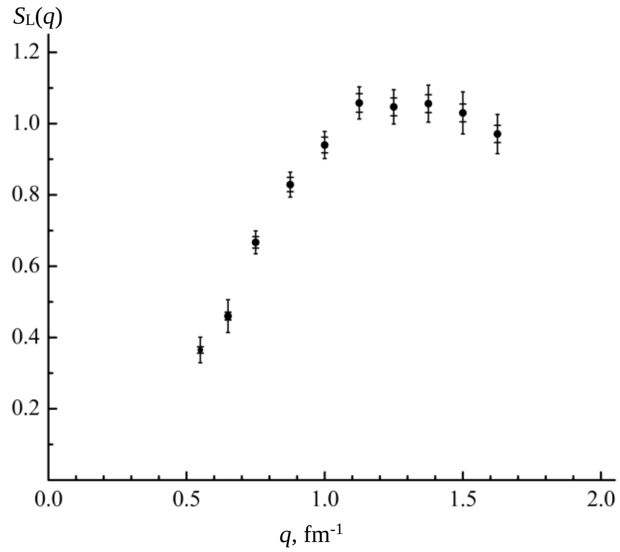

Fig. 2. Experimental Coulomb sum values of ^{7}Li. The error bars at the points represent the sum of systematic and statistical uncertainties. The systematic uncertainties are shown by a wide cap on the line of the summary error bar.

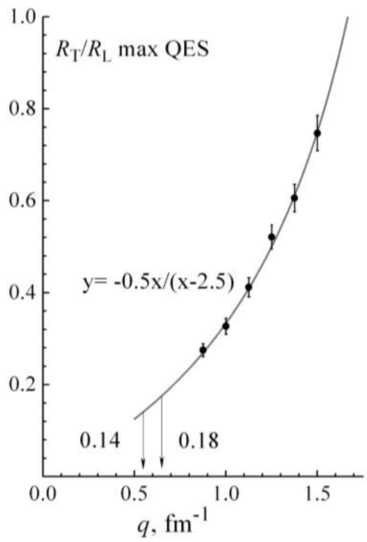

Fig. 3. Extrapolation of experimental R_T/R_L ratio values at the QES peak maximum to q = 0.55, 0.65 fm^{-1}. The arrows show the momentum transfer value, to which the extrapolation is carried out, and nearby, the obtained R_T/R_L ratio values are.

As a result of the performed revisions, the Coulomb sum values of ^{7}Li have changed only slightly. Their final values are presented in Fig. 2. The same Figure shows the estimated $S_L(q)$ values at

q = 0.55 and 0.65 fm^{-1}. These values were obtained from the measurements at E_0 = 160 MeV and θ = = 40.5 and 49°, i.e., at the conditions when the contribution from the longitudinal response function prevails in the measured cross-sections. Since the accelerator LUE-300 could not provide a stable electron beam of energy below 100 MeV, the spectrum measurements at large scattering angles and at q < 0.75 fm^{-1} were impossible to perform. Therefore, the $R_T(q, \omega)$ data, required for determining the $R_L(q, \omega)$ values, were derived from the extrapolation of the R_T/R_L ratios determined at higher momentum transfers (Fig. 3). The obtained in this way $S_L(q)$ values exhibit moderate accuracy, characterized by ~ 10 % errors.

5. Data analysis with consideration of the form factor of the ^{7}Li ground state

The experimental values of $F_{el}^2(q)$ for the ^{7}Li nucleus were reported in the published papers [9, 13]. However, those form factors were obtained about 50 years ago, and they correspond to the equipment capabilities and data processing experience of that time. Therefore, when turning to the data of [9, 13], their revision and, possibly, some correction should be made. In view of this, we have analyzed the works [9, 13], and on the basis of the analysis carried out, made the following corrections for the mentioned data.

First. The momentum transfers, at which the experimental form factors had been obtained in those works, in the present work were transformed by using expression (3) into the effective momentum transfers.

Secondly. Since the weak point in many works on processing of electron scattering experiments lay in low accuracy of data normalization (data absolutization), it was necessary to verify the normalization of the form factors under discussion, and in case of necessity to renormalize the data. The realization of this procedure is based on the definition and properties of the form factor as a physical quantity. Thus, the form factor of the nuclear ground state corresponds to the condition that at $q \to 0$, $F_{el}^2(q)$ tends to 1. Do the experimental form factors comply with this condition? Let us make the $f(a_i, q)$ function with the variable parameters a_i, and which corresponds to the above-mentioned condition, to fit the experimental data. If these data have been normalized incorrectly, then the function, which approximates their, at $q \to 0$ will tend to a certain number other than unity. Whereas the function $f(a_i, q)$, being at $q = 0$ “bound” to unity, will be “skewed” at its fitting to those data, thereby deteriorating the minimum χ^2 value.

Let us take the function $\varphi = k f(a_i, q)$ with the free parameters k, a_i and fit it to the experimental $F_{el}^2(q)$. The free parameter k, removes the requirement $\varphi(q = 0) = 1$. If this fit gives us $k \pm \Delta k = 1$, it means that the function $f(a_i,q)$ approximates the data, which have been normalized correctly to an accuracy of $\Delta k/k$. If, however, we have $k \pm \Delta k \neq 1$, then with the use of this parameter we can renormalize the data, and their new values will be $F'^2_{el}(q) = F_{el}^2(q)/k$. [2]

As functions that can be used to approximate the experimental form factors, the authors of work [13] have used $f_1(a_i, q)$, i.e., the expression corresponding to a simplified harmonic-oscillator shell model, and also, $f_2(a_i, q)$ being the expansion in powers of q^2. We also made use of these expressions in order to calculate the parameters k_1 and k_2 in the two cases, respectively. As a result, we obtained $k_1 = 0.936 \pm \pm 0.011$ and $k_2 = 0.948 \pm 0.012$. Using the arithmetic mean value of k_1 and k_2, we have renormalized the experimental nuclear form factors and the curve that approximated them. The data from [9] were corrected in a similar manner. Fig. 4 shows the form factors obtained in this way, and the function describing them. It can be seen that for q getting closer to 2 fm^{-1}, the form factor is practically set to zero. This can be verified numerically: the integral of the form factor from q = 0 to q = 2 fm^{-1} increments its value in the region q = 0 ÷ 6 fm^{-1} by $7 \cdot 10^{-4}$ of its magnitude.

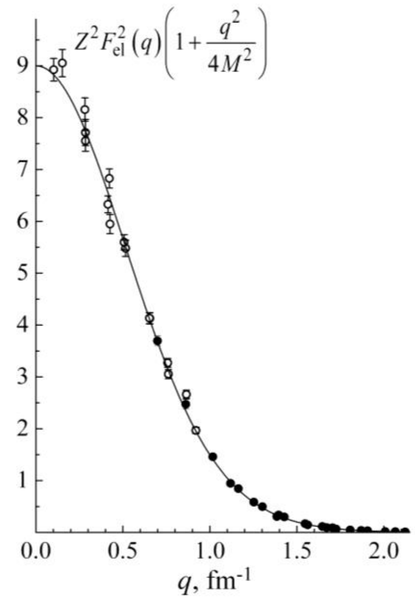

Fig. 4. Subintegral function of the first integral in Eq. (1) after data renormalization. $F_{el}(q)$ is the ground-state form factor of ^{7}Li. Open circles show the data from [13], full circles – data from [19].

[2] Unlike paper [9], the authors of work [13] applied a similar method of finding the normalization factors. However, the use of these factors without applying of the effective momentum transfers may introduce the error of several percent into the integral I_1.

On minimization of χ^2, besides the values of variable parameters a_i, their errors are also found, Δa_i. Hence, the statistical error of the integral will be

$$\int \left[\sum \left(\Delta a_i \cdot \partial f\left(a_i, q\right) / \partial a_i \right)^2 \right]^{1/2} dq .$$

As regards the systematic inaccuracy of the integral, it can be estimated only proceeding from the difference between the values of the k_1 and k_2 parameters, or from the difference between the integrals over the functions f_1 and f_2.

Then, using the functions that approximate the renormalized form factor values, we calculate the integral I_1 from the first component of Eq. (1). Considering the insignificance of the contribution to the integral I_1 from the bracketed multiplier that enters into its subintegral function, all the aforesaid about the integral without the brackets can be considered as referring to the integral I_1. As a result, we find the numerical value of the first component of Eq. (1), and multiplying by e^2/π we transform it to the MeV units

$$I_1 = 2.770 \pm 0.036 \pm 0.014 \text{ MeV}.$$

Here, the first uncertainty is statistical, the second one is systematic. Hereafter, the other numerical results will be presented in the same form.

6. The second integral of the equation for the nuclear Coulomb energy

The subintegral functions of the integral I_2 from Eq. (1) can be represented in the form as shown in Fig. 5.

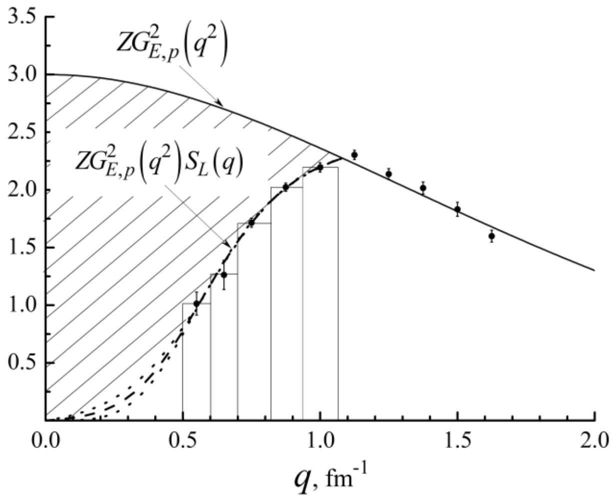

Fig. 5. Subintegral function of the second integral in Eq. (1). The shaded area between the upper solid line and the dashed line passing through the histogram corresponds to the integral value. The uncertainties at the experimental values are statistical.

In this representation, the integral value is determined by the shaded area between the functions $A(q) = Z \cdot G_{E,p}{}^2(q^2)$ and $B(q) = Z \cdot G_{E,p}{}^2(q^2) \cdot S_L(q)$. It is evident from the Figure that the upper limit of integral is determined by the point, where these two functions converge, i.e., at $q = 1.1$ fm^{-1}.

For the integral over the function $A(q)$, which we denote by I_A , the values of the form factors $G_{E,p}(q^2)$ and $\Delta G_{E,p}(q^2)$ are taken from [14]. As a result, we obtain

$$I_A = 1.3347 \pm 0.0053 \text{ MeV}.$$

We represent the integral over the function $B(q)$ (denoted by I_B) as the histogram area, where the width of the bins are not the same (see Fig. 5). The width of the i-th bin is designated by D_i, and the statistical uncertainty of $S_L(q_i)$ is indicated by $\Delta_{stat}S_L(q_i)$. As a result, the integral and its statistical uncertainty $\Delta_{stat}I_B$ will take the form

$$I_B = Z \sum_{i=1}^{i=6} G^2_{E,p}\left(q_i^2\right) S_L\left(q_i\right) D_i ,$$

$$\Delta_{stat} I_B = Z \sqrt{\sum_{i=1}^{i=6} \left[G^2_{E,p}\left(q_i^2\right) \Delta_{stat} S_L\left(q_i\right) D_i \right]^2}. \quad (6)$$

As may be seen from Fig. 5, at $q < 0.5$ fm^{-1}, where the histogram bin $i = 1$ should be, there are no experimental data. It may be inferred from the Figure that with decrease in the momentum transfer, the $S_L(q)$ value also rapidly decreases, and it can be extrapolated to lower momentum transfers. However, since the Coulomb sum describes nuclear reactions, but at $q = 0$ there can be no such reactions, then it is clear that $S_L(0) = 0$. So, if using this value, the function $S_L(q)$ can be interpolated rather than extrapolated, and this should give a more exact result.

Nine different functions within a few ranges of momentum transfers (from $q = 0 \div 0.875$ fm^{-1} up to $q = 0 \div 1.125$ fm^{-1}) were investigated with the aid of the software package Origin Pro 8.5. Here, both the χ_i^2 value per degree of freedom and the momentum transfer band width, at which the χ_i^2 value was minimized, served as the criteria in selecting the optimum interpolation function. In this approach, the Boltzmann ($\Phi_B(q)$) and the Logistic ($\Phi_L(q)$ functions were chosen: the acceptable χ_i^2 value being in the maximum fit range of $q = 0 \div 1.125$ fm^{-1}. It is interesting that these functions are very close at $q = 0.550 \div 1.125$ fm^{-1}, but at $q = 0.25$ fm^{-1} they become strongly divergent (see Fig. 5). In order to reveal the influence of $S_L(q)$ values at $q = 0.55$ and 0.65 fm^{-1} on the interpolation some fittings were performed, where the error bars of these two points were varied by a factor of 1.5. First, the error bars were increased, then the error bars were decreased. In both cases, the effect on the fit result was negligible.

Eventually, the midline between the Boltzmann function and the Logistic function was taken as the line of interpolation, and the functions themselves were taken for the error corridor at $q < 0.5$ fm^{-1}. In the problem under consideration, the area under the function $S_L(q)$ was required on the interval $q = 0 \div$ $\div\, 0.5$ fm^{-1}. Therefore, we took the arithmetic mean value of the integrated Logistic and Boltzmann functions on this interval as the area of the first histogram bin S_1. The error of the obtained value was about $0.16S_1$. After substitution of the data into Eq. (6) we find

$$I_B = 0.493 \pm 0.012 \pm 0.012 \text{ MeV}.$$

Since the integral $I_2 = I_A - I_B$, then we have

$$I_2 = 0.842 \pm 0.013 \pm 0.012 \text{ MeV},$$

$$E_{coul} = 1.928 \pm 0.038 \pm 0.026 \text{ MeV}.$$

7. Discussion and conclusions

For the analysis of the results of the present study we need the Coulomb energy data for the ^{7}Li nucleus, and also, for ^{6}Li and ^{4}He nuclei. These data are given in the Table.

Nucleus	E_{coul}, MeV	I_1, MeV	I_2, MeV
^{7}Li	1.928 ± 0.064	2.770 ± 0.050	0.842 ± 0.025
^{6}Li*	1.60 ± 0.10	2.45 ± 0.05	0.85 ± 0.09
^{4}He**	1.02 ± 0.10	1.98 ± 0.08	0.96 ± 0.06

* [2, 3].
** [5], more precise in [23, page 215].

1. The Coulomb energy of nucleus is the higher the smaller are the interproton distances. So, the ratio $E_{coul}(^7\text{Li})/E_{coul}(^6\text{Li}) = 1.205 \pm 0.085$ is in complete concordance with the fact that the nucleus ^{7}Li is smaller than the nucleus ^{6}Li. The same conclusion has followed from the experimental data on elastic electron scattering by the nuclei of lithium isotopes [9].

2. In the consideration of the research results based on Eq. (1), not only the calculated total Coulomb energy E_{coul}, but also the numerical values of the integrals I_1 and I_2 entering into the equation are of importance.

Physically, the integral I_1 is the Coulomb energy of the electric charge, the spatial distribution of which is displayed by the form factor of the nuclear ground state. This is the Coulomb energy, which is generally attributed to the atomic nucleus, as it was first done in the liquid-drop nuclear model and, with time, was refined through the introduction of more realistic models of charge distribution in the nucleus.

The physical significance of the integral I_2 is the Coulomb energy decrease due to the wave functions overlapping of the protons that constitute the nucleus. In Eq. (1), this integral can be considered as a correction that takes into account the influence of mutual arrangement of protons in the nucleus on the Coulomb energy of this nucleus.

^{7}Li shows a high degree of clusterization and consists of α- and *t*-clusters, with the spacing between them greater than the distance between protons in the α-cluster. Therefore, we can assume that the integral I_2 of the ^{7}Li nucleus is completely determined by the overlap of the wave functions of the protons belonging to the α-cluster.

3. The value of the integral I_2 determines the degree of the wave functions overlapping of the two protons, and it is the larger, than smaller distance between these protons. Therefore, if the integral I_2 in the equation for ^{4}He differs from the integral I_2 for the case of the ^{7}Li nucleus, this indicates that the distance between protons in the ^{4}He nucleus and in the α-cluster belonging to ^{7}Li also differs. Let us compare the experimental values of I_2: $I_2(^7\text{Li})/I_2(^4\text{He}) = 0.87 \pm 0.06$. Thus, the obtained ratio indicates that the distance between protons in the α-cluster is greater than in the ^{4}He nucleus. This means that, within the two-cluster consideration of the ^{7}Li nucleus (α + t), the size of the α-cluster should be larger than that of the ^{4}He nucleus.

4. The ^{6}Li nucleus is also strongly clusterized, and comprises the α-cluster, therefore it must be supposed that in this case also the integral I_2 in the equation E_{coul} is determined by protons of the α-cluster. However, a more detailed consideration of the case of the ^{6}Li nucleus is difficult due to the large errors in the experimental values of E_{coul} and I_2 of this nucleus.

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КУЛОНОВА СУМА ЯДРА ^{7}Li

Отримано експериментальні значення поздовжньої функції відгуку ядра ^{7}Li та на їхній базі знайдено значення кулонової суми в діапазоні переданих імпульсів від 0,550 до 1,625 фм$^{-1}$. Використовуючи знайдені значення кулонової суми, було визначено повну кулонову енергію ядра ^{7}Li. Результат порівняння кулонової енергії ядра ^{7}Li з кулоновими енергіями ядер ^{6}Li та ^{4}He а) узгоджується із виявленою в експерименті з пружного розсіяння електронів аномалію – середньоквадратичний радіус ядра ^{7}Li менший за середньо-квадратичний радіус ядра ^{6}Li; б) приводить до висновку, що, у рамках двокластерної моделі ядра ^{7}Li (α + t), розмір α-кластера має бути більшим за розмір ядра ^{4}He.

Ключові слова: розсіяння електронів, ^{7}Li, поздовжня функція відгуку, кулонова сума, кулонова енергія, кластери.

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КУЛОНОВСКАЯ СУММА ЯДРА ^{7}Li

Получены экспериментальные значения продольной функции отклика ядра ^{7}Li и на этой базе найдены значения кулоновской суммы в диапазоне переданных импульсов от 0,550 до 1,625 фм$^{-1}$. С помощью полученных значений кулоновской суммы определена полная кулоновская энергия ядра ^{7}Li. Результат сравнения кулоновской энергии ядра ^{7}Li с кулоновскими энергиями ядер ^{6}Li и ^{4}He а) согласуется с обнаруженной в эксперименте по упругому рассеянию электронов аномалией – среднеквадратичный радиус ядра ^{7}Li меньше, чем среднеквадратичный радиус ядра ^{6}Li; б) приводит к выводу, что, в рамках двухкластерной модели ядра ^{7}Li (α + t), размер α-кластера должен быть больше, чем размер ядра ^{4}He.

Ключевые слова: рассеяние электронов, ^{7}Li, продольная функция отклика, кулоновская сумма, кулоновская энергия, кластеры.

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