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DIFFRACTION SCATTERING OF ^{6}Li IONS FROM ATOMIC NUCLEI

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Using formalism of diffraction approximation, the elastic scattering of ^{6}Li ions from ^{28}Si, ^{40}Ca, and ^{90}Zr nuclei has been investigated at an incident energy of 35 and 53 MeV per nucleon. We have considered ^{6}Li nucleus as a weakly-bound one that consists of two charged clusters, there are deuteron and alpha-particle. The calculated angular distributions of cross sections satisfactorily fit corresponding experimental data.

1. Inrtoduction

Diffraction phenomena in nuclear reactions arise when the De Broigle wave length of projectile become less as compared with a dimension of interaction region (that is valid at an incident energy of 10 - 15 MeV/nucleon and more). The nuclear diffraction, in contrast to the optical one, characterized by wide variety of inelastic diffraction processes taking place at collisions of complex particles with nuclei. The absence of free adjustable parameters is one of the advantages of the diffraction model that allows us to introduce the nuclear characteristics (such as diffuseness of nuclear surface, nonsphericalness, spin etc.) naturally [1, 2]. The diffraction approach was found as successful for theoretical description of nuclear reactions with participation of neutron-riched and exotic nuclei that are intensively investigated lately [3 - 6]. The first studies of deuteron [7, 8] showed that diffraction processes with deuterons can be theoretically explained by taking into account both the nuclear interaction and the Coulomb one. In those early works the Coulomb part of amplitude expressed only in terms of impulse approximation. To describe observables exactly, it was necessary to remove logarithmic divergences appearing in that part of amplitude which corresponds to double scattering of deuteron clusters. This problem was successfully solved by [3]. Supposing that an incident weakly-bound two-cluster nucleus consists of the charged cluster and the neutral one, they developed the general formalism allows to describe a diffraction scattering and dissociation of such a nuclei colliding with non-spin ones. Further, this approach was successfully used for description of ^{2}H scattering from set of medium and heavy targets [9, 10]. Besides, the method [3] was generalized for the case when an incident nucleus has two charged clusters [11] that allow us to describe experimental cross section of ^{6}Li scattering from ^{12}C at 156 MeV. The formalism [11] was also successfully employed for description of ^{3}He elastic scattering from ^{40}Ca at 130 MeV and energy spectra of deuterons from ^{28}Si(^{3}He,dp) reaction at an incident energy of 52 MeV [12]. In proceeding of mentioned works, the present paper is devoted to investigation of ^{6}Li diffraction scattering from medium nuclei.

2. Formalism

The ^{6}Li nucleus considered in our calculations as weakly-bound one that consists of deuteron and alpha-particle, all our following calculations are carried out in non-spin approximation and for the center-of-mass system. Assuming that targets have spherical form, let us start from the general expression for scattering amplitude ($h = c = 1$) [3]

$$G_0(\vec{q}) = \frac{k}{k_1} f_1(\vec{q})\,\Phi_0(-\beta_2\vec{q}) + \frac{k}{k_2} f_2(\vec{q})\,\Phi_0(\beta_1\vec{q}) + $$

$$+\frac{ik}{2\pi k_1 k_2}\int d^{(2)}\vec{s}\,\Phi_0(\vec{s})\,f_1(\beta_1\vec{q}-\vec{s})\,f_2(\beta_2\vec{q}+\vec{s})\,, \quad (1)$$

where $\vec{q}$ is the vector of momentum transfer, k is the momentum of incident nucleus, k_j is the momentum of j-th cluster $(j = 1, 2)$, $\beta_j = A_j/(A_1+A_2)$, A_j is the corresponding mass number. The scattering amplitude $f_j(\vec{q})$ in Eq. (1) is the Fourier transform of cluster-nucleus profile function $\omega_j(\vec{s}_j)$, where $\vec{s}_j$ is the impact parameter vector

$$f_j(\vec{q}) = \frac{ik_j}{2\pi}\int d^{(2)}\vec{s}_j\, e^{i\vec{q}\vec{s}_j}\,\omega_j(\vec{s}_j)\,. \quad (2)$$

Φ_0 in Eq. (1) is the incident nucleus formfactor that expressed in the following way

$$\Phi_0(\vec{s}) = \int d^{(3)}\vec{r}\, e^{i\vec{s}\vec{r}}\,|\varphi_0(\vec{r})|^2\,, \quad (3)$$

where $\varphi_0(\vec{r})$ is the ^{6}Li wave function desribing relative motion of deuteron (j = 1) and ^{4}He (j = 2).

Considering a targets as spherical (that is valid for even-even nuclei), let's write the profile function $\omega_j(\vec{s}_j) \equiv \omega_j(s_j)$ in a form [3, 6]

$$\omega_j(s_j) = \left(1-\Theta(s_j)\,e^{2i\eta_j(s_j)}\right)\left(1 + e^{\frac{s_j-R_j}{\Delta}}\right)^{-1}, \quad (4)$$

where Θ is the Heaviside function, $\eta_j(s_j) = n_j \ln k_j s_j$ is the Coulomb scattering phase [1], n_j is the Sommerfeld parameter, $R_j = r_0\left(A_j^{1/3}+A^{1/3}\right)$ is the radius of interaction between j-th cluster of incident nucleus and a target containing A nucleons, Δ is the parameter of target surface diffuseness.

Substituting Eq. (4) into Eq. (2), after a number of conversions, we obtain [3]

$$G_0(\vec{q}) = G_0^N(\vec{q}) + G_0^{Z,1}(\vec{q}) + G_0^{Z,2}(\vec{q}), \quad (5)$$

where $G_0^N(\vec{q})$ is the nuclear component of amplitude (1)

$$G_0^N(\vec{q}) = ik\left\{\Omega_1(q)\Phi_0(-\beta_2\vec{q}) + \Omega_2(q)\Phi_0(\beta_1\vec{q}) - (2\pi)^{-1}\int d^{(2)}\vec{s}\,\Phi_0(\vec{s})\,\Omega_1(|\beta_1\vec{q} - \vec{s}|)\,\Omega_2(|\beta_2\vec{q} + \vec{s}|)\right\},$$

$$\Omega_j(q) = R_j^2\int_0^\infty d\xi\,\xi\,J_0(qR_j\xi)\left(1+ e^{\frac{(\xi-1)R_j}{\Delta}}\right)^{-1}. \quad (6)$$

$G_0^{Z,1}(\vec{q})$ and $G_0^{Z,2}(\vec{q})$ are the Coulomb parts of $G_0(\vec{q})$ that correspond to single and double scattering of projectile clusters [11, 12]

$$G_0^{Z,1}(\vec{q}) = g_{11}(q)\Phi_0(-\beta_2\vec{q}) + g_{12}(q)\Phi_0(\beta_1\vec{q}), \quad (7)$$

$$g_{1j}(q) = -\frac{2kn_j}{q^2}g_j(q), \quad n_j = \alpha_f Z_j Z\sqrt{\frac{m(A_1+A_2)}{2E}},$$

$$g_j(q) = \left(\frac{2}{qR_j}\right)^{2in_j}\frac{\Gamma(1 + in_j)}{\Gamma(1 - in_j)}-qR_j\int_0^1 dx\,x^{2in_j}J_1(qR_jx),$$

$$G_0^{Z,2}(\vec{q}) = g_{21}(\vec{q}) + g_{22}(\vec{q}), \quad (8)$$

$$g_{2j}(\vec{q}) = \frac{kn_jR_j}{\sqrt{2\pi\lambda}}\int_0^\infty dt\int_0^\pi d\vartheta\sin\vartheta\,\Phi_0(|\vec{t} - \beta_j\vec{q}|)\,e^{-\frac{t^2\sin^2 J}{16\lambda}}I_0\left(\frac{t^2\sin^2\vartheta}{16\lambda}\right)\frac{J_1(R_j|\vec{t} - \vec{q}|)}{|\vec{t} - \vec{q}|}g_j(t),$$

where $\alpha_f \cong 1/137$, $Z_j(Z)$ is the charge number of j-th cluster (target), m is the nucleon mass, E is an initial kinetic energy of projectile in lab system [13], $\lambda = 3/(16\langle r^2\rangle)$ [3], $\langle r^2\rangle^{1/2}$ is the experimental root mean square radius of ^{6}Li.

In the present work we did not investigate an influence of $\varphi_0(\vec{r})$ choice on values and behaviour of reaction cross sections since this question was studied in detail previously (see [9 - 11, 14]). In those papers it was showed that best agreement with an experimental data can be reached under the assumption that $\varphi_0(\vec{r})^o\varphi_0(r)$ has Hulten form

$$\varphi_0(r) = \sqrt{\frac{\alpha\beta(\alpha+\beta)}{2\pi(\beta-\alpha)^2}}\,\frac{e^{-\alpha r}-e^{-\beta r}}{r}, \quad (9)$$

where $\alpha = \sqrt{2\beta_2 A_1 m\varepsilon}$ [5], ε is the binding energy of ^{6}Li with respect to disintegration $^6Li(\gamma,\alpha)^2H$ (value $\varepsilon \simeq 1.5$ MeV was derived from the photonuclear reaction threshold). Using Eq. (9), the value of β can be derived from the relation [15]

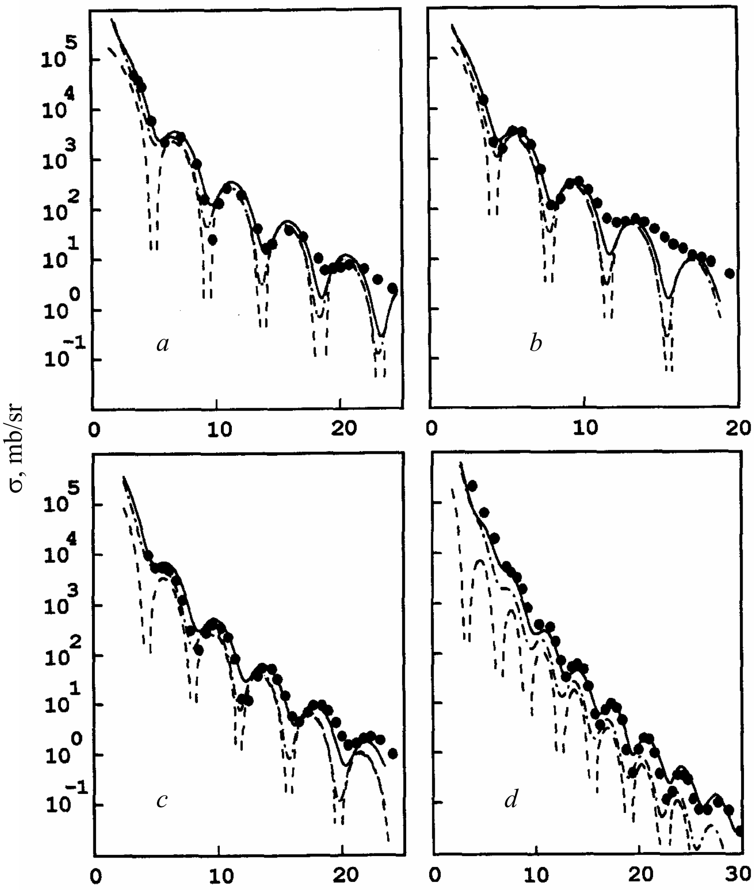
$$\langle r^2\rangle^{1/2} = \frac{1}{2}\left\{\int d^{(3)}\vec{r}\,r^2\,|\varphi_0(r)|^2\right\}^{1/2}. \quad (10)$$

3. Results and discussion

The differential cross sections were calculated according to known formula [1]

$$\sigma \equiv \sigma(\theta) = |G_0(\vec{q})|^2, \quad q = 2k\sin(\theta/2),$$

where θ is the scattering angle. The value of $\langle r^2\rangle^{1/2} \cong 2.45$ fm was taken from [16], the values α and β were derived from the relations (9) - (10), there were $\alpha = 0.311\ \text{fm}^{-1}$ and $\beta = 0.406\ \text{fm}^{-1}$. The computed values of σ for ^{6}Li scattering from ^{28}Si, ^{40}Ca, and ^{90}Zr are shown in the Figure. There are the "pure nuclear" cross section $\sigma_0 = |G_0^N(\vec{q})|^2$ (dashed curves), the "impulse Coulomb's approximation" cross section $\sigma_1 = |G_0^N(\vec{q}) + G_0^{Z,1}(\vec{q})|^2$ (dot-dashed curves), and $\sigma_2 = |G_0^N(\vec{q}) + G_0^{Z,1}(\vec{q}) + G_0^{Z,2}(\vec{q})|^2$ (solid curves). All calculated values of σ_2 were fitted with the experiment [17, 18] according to criterion of χ^2, ones were received using the following sets of parameters: a) $r_0 = 1.34$ fm, $\Delta = 0.28$ fm; b) $r_0 = 1.29$ fm, $\Delta = 0.31$ fm; c) $r_0 = 1.34$ fm, $\Delta = 0.28$ fm; d) $r_0 = 1.31$ fm, $\Delta = 0.57$ fm. Note that these values are approximately equal to corresponding ones derived from optical model analysis ([17, 18] and refs. therein).



Differential cross sections of ^{6}Li scattering from ^{28}Si (*a*, *b*), ^{40}Ca (*c*), and ^{90}Zr (*d*).
The projectile energies are 210 MeV (*a*, *c*, *d*) and 310 MeV (*b*).
Explanatory notes to the calculated curves are in the text. Experimental data (points) were taken from [17, 18].

Comparing the theoretical results with the experimental data, one can be conclude that including of $G_0^{Z,2}(\vec{q})$ (the double scattering Coulomb's term) into the reaction amplitude (5) leads to significant improvement of agreement for all considered cases a) - d). In general, it can be stated that correct "switching on" of the Coulomb interaction in $G_0(\vec{q})$ leads to increasing of absolute values of $\sigma(\theta)$ and filling of secondary diffraction minima. Some disagreement for angle region $\theta > 10^{\circ}$ in Fig. *a*), *b*) can be explained by an experimental inexactness such as an admixture of inelastic events. Similar discrepancy was established also in [3] that authors explain as a possible excitation of collective state 2^+ of ^{28}Si at 1.78 MeV.

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ДИФРАКЦІЙНЕ РОЗСІЯННЯ ІОНІВ ^{6}Li АТОМНИМИ ЯДРАМИ

В. І. Ковальчук

З використанням формалізму дифракційного наближення досліджено пружне розсіяння іонів ^{6}Li ядрами ^{28}Si, ^{40}Ca, ^{90}Zr при енергіях падаючих частинок 35 і 53 МеВ на нуклон. Ядро ^{6}Li розглядалося як слабкозв’язане, яке складається з двох заряджених кластерів – дейтрона та альфа-частинки. Розраховані кутові розподіли перерізів реакції задовільно узгоджуються з відповідними експериментальними даними.

ДИФРАКЦИОННОЕ РАССЕЯНИЕ ИОНОВ ^{6}Li АТОМНЫМИ ЯДРАМИ

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С использованием формализма дифракционного приближения изучено упругое рассеяние ионов ^{6}Li ядрами ^{28}Si, ^{40}Ca, ^{90}Zr при энергиях падающих частиц 35 и 53 МэВ на нуклон. Ядро ^{6}Li рассматривалось как слабосвязанное, состоящее из двух заряженных кластеров – дейтрона и альфа-частицы. Рассчитанные угловые распределения сечений реакции удовлетворительно согласуются с соответствующими экспериментальными данными.

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