

## INTERACTION POTENTIAL BETWEEN TWO AXIALLY SYMMETRIC NUCLEI

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The simple approach for the evaluation of the interaction potential between two deformed axial-symmetric nuclei is proposed. The potential energy of deformed arbitrary-oriented nuclei is discussed in detail. Properties of entrance-channel potential for fusion reactions  $^{22}\text{Ne} + ^{248}\text{Cm}$  and  $^{26}\text{Mg} + ^{248}\text{Cm}$  leading to super-heavy elements are considered.

### 1. Introduction

There are many various shapes of nuclei. Many nuclei are well-deformed in the ground-state. Therefore reactions between deformed nuclei have been often studied both experimentally and theoretically [1 - 10]. Such nuclear reactions have been used in various laboratories for the production of heavy and super-heavy elements [9, 10]. The radiochemistry of super-heavy elements is often done in reactions between heavy actinide targets and deformed light projectiles [10].

It has been show that the sub-barrier fusion of spherical and well-deformed nuclei in the ground-state is strongly enhanced by deformation [1, 4 - 8]. The enhancement of sub-barrier fusion of two well-deformed nuclei induced by deformation of both nuclei is very prominent [6].

The interaction potential between nuclei consists of nuclear, Coulomb and centrifugal parts [2, 3]. The Coulomb interaction energy is related to a six-dimensional integral [2, 7, 11] while the nuclear part of nucleus-nucleus potential is obtained by six- or three-dimensional integrals in the framework of various models [2, 12 - 14]. Evaluation of these integrals is intricate numerical problem especially in the case, when both nuclei are deformed in the ground-states.

Therefore the Coulomb interaction of two deformed nuclei is often approximated by various expressions of different accuracy [4, 6]. As a rule the first order corrections on the deformation parameters to the Coulomb interaction of two point nuclei are taken into account [6]. Some terms of the second-order corrections for specific orientation of two deformed nuclei are given in Ref. [4]. Below we derive expression contained all second-order terms on deformation for the Coulomb interaction between two axial-symmetric deformed nuclei, which are arbitrary oriented.

The nuclear part of interaction between two spherical nuclei is evaluated by using semi-microscopic approximation [12, 13]. The expression for the nuclear part of nucleus-nucleus interaction is obtained by fitting the 7140 potentials between 119 spherical or near spherical nuclei along the

$\beta$ -stability line from  $^{16}\text{O}$  to  $^{212}\text{Po}$  [12]. The heights and radii of experimental fusion barriers are well described by this potential for various pairs of interacting nuclei [12, 14, 15]. Applying the proximity theorem [16] we obtain the nuclear part of the potential between two deformed nuclei by modifying the expression from Ref. [12]. Note that the values of potential evaluated by applying the proximity theorem for deformed nuclei and double-folding method agree fairly well [17, 18].

We analyze in detail the dependence of the nucleus-nucleus potential on various orientations of deformed nuclei in the framework of our approach.

The expression for the Coulomb interaction between two axial-symmetric deformed nuclei arbitrary oriented is presented in the section 2. Our approximation for the evaluation of the nuclear part of the potential is discussed in section 3. Main features of nucleus-nucleus interactions for arbitrary oriented deformed nuclei and conclusion are given in section 4.

### 2. The Coulomb interaction of two axial-symmetric nuclei

The Coulomb interaction of two nuclei at distances between their mass centres  $R$  is

$$V_C(R) = \int \frac{\rho_1(r_1)\rho_2(r_2)}{|R + r_2 - r_1|} dr_1 dr_2, \quad (1)$$

where  $\rho_i(r_i)$  is the proton density in the nucleus  $i$ ,  $i=1,2$ . The vectors  $r_i$  are determined in the coordinate systems  $O_1$  and  $O_2$  as illustrated in Fig. 1. The origins of these coordinate systems are connected to the mass centres of corresponding nuclei.

We consider interaction of two arbitrary oriented axial-symmetric nuclei. We choose the directions of coordinates  $X_i$  and  $Y_i$  in such a way that an axial-symmetry axis of the first nucleus is located in the plane  $X_1 Z_1$  without the loss of generality. The angle between the axial-symmetry axis of the first nucleus and  $Z_1$  axis is  $\Theta_1$ . Then angles  $\Theta_2$  and  $\Phi$  determine the orientation of the axial-symmetry axis of the second nucleus in the coordinate system  $O_2$ .

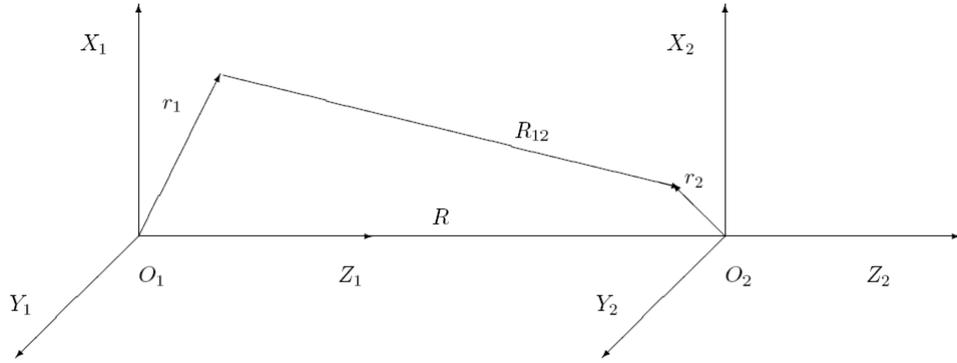


Fig. 1. The coordinate systems related to the mass centres of corresponding nuclei.

For the sake of simplicity we propose the step proton density distributions in both nuclei

$$\rho_i(r) = \rho_{i0}\theta(R_i(\mathcal{G}^i) - r), \quad (2)$$

where  $\theta(x)$  is the step function and

$$R_i(\mathcal{G}^i) = R_{i0} \left[ 1 + \sum_{l \geq 2} \beta_{il} Y_{l0}(\mathcal{G}^i) \right], \quad (3)$$

is the distance of the deformed nuclear surface from the origin in a intrinsic coordinate system of the nucleus  $i$ . Here  $R_{i0}$  and  $\beta_{il}$  are the radius and the deformation parameters respectively. Note that the axis  $Z_i$  of the intrinsic coordinate system  $O_i$  coincides with the axial-symmetry axis of the nucleus  $i$ .

The  $\Theta_i$ ,  $\Theta_2$  and  $\Phi$  are the Euler angles, which define the rotation transformations between intrinsic systems  $O_{1'}$  and  $O_{2'}$  and laboratory systems  $O_1$  and  $O_2$ , respectively. The transformation of spherical harmonic functions  $Y_{lm}(\theta, \varphi)$  at rotations is described by the Wigner  $D$ -functions  $D_{m, m_2}^l(\Phi_1, \Theta, \Phi_2)$  [19]. As the result the distances of the deformed nuclear surfaces from the origin of the coordinate system  $O_1$  and  $O_2$  are correspondingly given as

$$R_1(\mathcal{G}_1, \varphi_1, \Theta_1) = R_{10} \left[ 1 + \sum_{l \geq 2} \beta_{1l} \sum_{m=-l}^l Y_{lm}(\mathcal{G}_1', \varphi_1') D_{m, 0}^l(0, \Theta_1, 0) \right], \quad (4)$$

$$R_2(\mathcal{G}_2, \varphi_2, \Theta_2, \Phi) = R_{20} \left[ 1 + \sum_{l \geq 2} \beta_{2l} \sum_{m=-l}^l Y_{lm}(\mathcal{G}_2', \varphi_2') D_{m, 0}^l(\Phi, \Theta_2, 0) \right]. \quad (5)$$

Using these expressions for rotational transformations and substituting (2) and (3) into (1) we can evaluate integrals in (1). We take into account linear and quadratic terms on the quadrupole ( $l=2$ ) deformation parameters and linear terms on high-

multipolarity ( $l \geq 3$ ) deformation parameters, because values of the ground state deformation parameters in stable nuclei are  $(\beta_2)^2 \approx \beta_{l \geq 3}$  as a rule [20]. Hence the Coulomb interaction of two axial-symmetric arbitrary-oriented nuclei is

$$V_C(R, \Theta_1, \Theta_2, \Phi) = \frac{Z_1 Z_2 e^2}{R} \left\{ 1 + \sum_{l \geq 2} [f_{1l}(R, \Theta_1, R_{10}) \beta_{1l} + f_{2l}(R, \Theta_2, R_{20}) \beta_{2l}] + f_2(R, \Theta_1, R_{10}) \beta_{12}^2 + f_2(R, \Theta_2, R_{20}) \beta_{22}^2 + f_3(R, \Theta_1, \Theta_2, R_{10}, R_{20}) \beta_{12} \beta_{22} + f_4(R, \Theta_1, \Theta_2, \Phi, R_{10}, R_{20}) \beta_{12} \beta_{22} \right\}, \quad (6)$$

where  $Z_1$  and  $Z_2$  are the number of protons in corresponding nuclei,  $e$  is the charge of proton,

$$f_{1l}(R, \Theta_i, R_{i0}) = \frac{3R_{i0}^l}{(2l+1)R^l} Y_{l0}(\Theta_i), \quad (7)$$

$$f_2(R, \Theta_i, R_{i0}) = \frac{6\sqrt{5}R_{i0}^2}{35\sqrt{\pi}R^2} Y_{20}(\Theta_i) + \frac{3R_{i0}^4}{7\sqrt{\pi}R^4} Y_{40}(\Theta_i), \quad (8)$$

$$f_3(R, \Theta_1, \Theta_2, R_{10}, R_{20}) = \frac{R_{10}^2 R_{20}^2}{R^4} v_0(\Theta_1, \Theta_2), \quad (9)$$

$$f_4(R, \Theta_1, \Theta_2, \Phi, R_{10}, R_{20}) = \frac{27R_{10}^2 R_{20}^2}{40\pi R^4} \left[ \cos(\Phi)^2 v_1(\Theta_1, \Theta_2) - \cos(\Phi) v_2(\Theta_1, \Theta_2) \right]. \quad (10)$$

Here  $v_i(a, b)$  are the functions of  $a$  and  $b$ . Note that  $v_{1(2)}(\Theta_1, \Theta_2) = 0$ , if  $\Theta_1 = 0$  or  $\Theta_2 = 0$ . (Functions  $v_i(a, b)$  are cumbersome; therefore explicit forms of them are omitted here for simplicity.) We see that the Coulomb interaction of two deformed axial-symmetric nuclei depends on orientation angles  $\Theta_i$ ,

$\Theta_2$  and  $\Phi$  and consists of terms proportional to  $\beta_{1l}$ ,  $\beta_{2l}$ ,  $\beta_{12}^2$ ,  $\beta_{22}^2$  and  $\beta_{12}\beta_{22}$ . Here and below we take into account the volume correction appearing in the second order on the deformation parameter.

The expression for the Coulomb interaction of two deformed axial-symmetric nuclei depending on  $\Theta_1$ ,  $\Theta_2$ ,  $\beta_{1l}$ ,  $\beta_{2l}$ ,  $\beta_{12}^2$  and  $\beta_{22}^2$  has been discussed in Ref. [4]. However additional terms containing  $\beta_{12}\beta_{22}$  are presented in our expression. Terms including  $\Phi$  have been ignored in all previously discussed expressions for the Coulomb interaction of two deformed nuclei.

### 3. The nuclear interaction of two axial-symmetric nuclei

The functional form of the nuclear interaction potential between spherical nuclei has discussed in Ref. [12]. This potential is obtained by using the semi-microscopic calculations of the interaction energy of two nuclei. The semi-microscopic interaction potentials for the distances around the touching point are systematically studied in the frozen-density approximation. The potential energy is evaluated in the Thomas-Fermi approximation extended to second-order gradient contributions. The (frozen) density of projectile and target nuclei are obtained from the Hartree-Fock-BCS calculations for the Skyrme force parametrization SkM\*. The resulting 7140 semi-microscopic potentials between the spherical nuclei around touching points have been evaluated. The expression for the potential is obtained by fitting these potentials around the touching points. The heights and radii of empirical fusion barriers are well described by this potential for various pairs of interacting nuclei [12, 14, 15].

The nuclear part of the potential depends on both the distance between surfaces of colliding nuclei  $d$  and the surface curvatures  $C_i$  of corresponding nuclei at the closest point [16]. Nuclear interaction potential of two spherical nuclei derived in Ref. [12] contains similar dependencies too. Therefore applying the proximity theorem [16] we may approximate the nuclear part of the potential between deformed nuclei as

$$V_n(d(R, \Theta_1, \Theta_2, \Phi, R_{10}, R_{20}, \beta_1, \beta_2)) \approx \frac{C_{10} + C_{20}}{C_1 + C_2} V_n^0(d^0(R_{\text{sph}}, R_{10}, R_{20})). \quad (11)$$

Here  $d^0(R_{\text{sph}}, R_{10}, R_{20}) = d(R, \Theta_1, \Theta_2, \Phi, R_{10}, R_{20}, \beta_1, \beta_2)$ ,  $V_n^0(d^0(R_{\text{sph}}, R_{10}, R_{20}))$  is the nuclear part of the interaction potential between two spherical nuclei

with radii  $R_{10}$  and  $R_{20}$  [12],  $d^0(R_{\text{sph}}, R_{10}, R_{20}) = R_{\text{sph}} - R_{10} - R_{20}$  is the distance between surfaces of two spherical nuclei at distance  $R_{\text{sph}}$  between their mass centres,  $C_{1(2)0} = 1/R_{1(2)0}$  are the surface curvatures of corresponding spherical nuclei,  $C_{1(2)}$  is the curvature of corresponding deformed nuclear surface at the point closest to the surface of another nucleus. Note that the curvature  $C_{1(2)}$  depends on both the orientation angle(s) and the deformation parameter  $\beta_{1(2)}$ . Equations for  $V_n^0(d^0(r, R_{10}, R_{20}))$  and  $R_{1(2)0}$  are given in [12].

Coordinates of closest points of surfaces and the curvatures of nuclear surfaces at these points are evaluated numerically. The distance between surfaces of colliding nuclei  $d(R, \Theta_1, \Theta_2, \Phi, R_{10}, R_{20}, \beta_1, \beta_2)$  at  $R \geq R_{12}^0$  is evaluated numerically, where

$$R_{12}^0 = R_{10} \left[ 1 + \sum_{l \geq 2} \beta_{1l} Y_{lo}(\mathcal{G}'_1) \right]_{|\mathcal{G}'_1=0} + R_{20} \left[ 1 + \sum_{l \geq 2} \beta_{2l} Y_{lo}(\mathcal{G}'_2) \right]_{|\mathcal{G}'_2=\pi}. \quad (12)$$

Note that the surfaces of colliding nuclei cannot intersect at  $R \geq R_{12}^0$ . At  $R \leq R_{12}^0$ , the nuclear surfaces can intersect and numerical evaluation of the intersect distance is not straightforward, therefore we approximate

$$d(R, \Theta_1, \Theta_2, \Phi, R_{10}, R_{20}, \beta_1, \beta_2) \approx (R - R_{12}^0) + d(R = R_{12}^0, \Theta_1, \Theta_2, \Phi, R_{10}, R_{20}, \beta_1, \beta_2) \quad (13)$$

for the sake of simplicity of the code, where  $d(R = R_{12}^0, \Theta_1, \Theta_2, \Phi, R_{10}, R_{20}, \beta_1, \beta_2)$  is the distance between surfaces at distance  $R = R_{12}^0$  at the same mutual orientation of the interacting nuclei.

Using this approach we can study various features of heavy-ion entrance-channel potential.

### 4. The interaction of axial-symmetric nuclei at various relative orientations

The fusion of heavy spherical nuclei with heavy deformed nuclei leading to very heavy systems depends on orientation of deformed nuclei [13, 21]. Therefore it is interesting to study interaction potential for systems  $^{64}\text{Zn} + ^{150}\text{Nd}$  and  $^{70}\text{Zn} + ^{144}\text{Nd}$ , which lead to the same compound-nucleus. Nucleus  $^{144}\text{Nd}$  is spherical [20, 22], while other nuclei are deformed. The values of ground-state quadrupole deformations are  $\beta_2(^{64}\text{Zn}) = 0,219$ ,  $\beta_2(^{70}\text{Zn}) = 0,045$  and  $\beta_2(^{150}\text{Nd}) = 0,243$  [20, 22]. So system  $^{64}\text{Zn} +$

+  $^{150}\text{Nd}$  is related to the interaction of two well-deformed prolate nuclei, while system  $^{70}\text{Zn} + ^{144}\text{Nd}$  is connected to the interaction of spherical and slightly-deformed nuclei. The result of potential evaluation for these systems in the framework of our model is presented in Figs. 2 and 3. The charges of

colliding nuclei and the total number on nucleons are the same for both systems; therefore the differences between potentials evaluated for different orientations and systems in Figs. 2 and 3 are mainly induced by deformation.

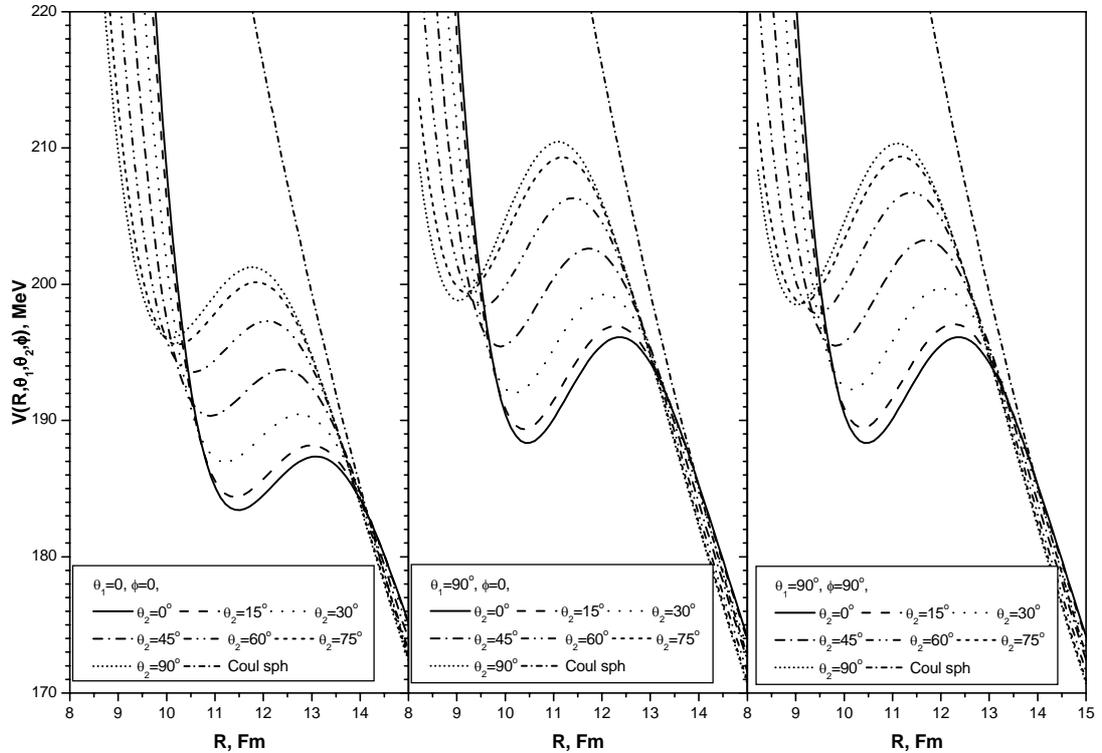


Fig. 2. Potentials between  $^{64}\text{Zn} + ^{150}\text{Nd}$  for various orientation of both  $^{64}\text{Zn}$  and  $^{150}\text{Nd}$ . The Coulomb potential between these nuclei, when shapes of both nuclei are considered spherical, is also presented for comparison.

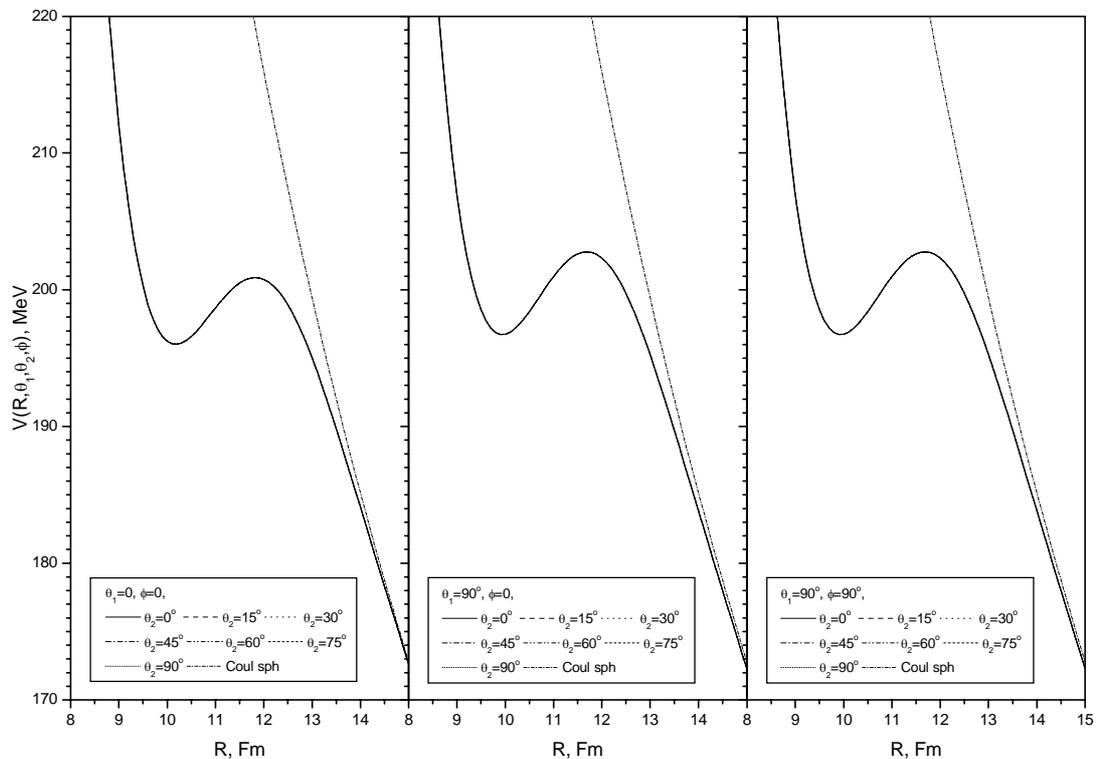


Fig. 3. Potentials between  $^{70}\text{Zn} + ^{144}\text{Nd}$  for various orientation of both  $^{70}\text{Zn}$  and  $^{144}\text{Nd}$ . The Coulomb potential between these nuclei, when both nuclei are considered spherical, is also presented for comparison.

The value of barrier for system  $^{64}\text{Zn} + ^{150}\text{Nd}$  for a tip-tip ( $\Theta_1 = 0^\circ$ ,  $\Theta_2 = 0^\circ$ ) orientation is the lowest. The highest barrier is obtained for a side-side ( $\Theta_1 = 90^\circ$ ,  $\Theta_2 = 90^\circ$ ) orientation of  $^{64}\text{Zn}$  and  $^{150}\text{Nd}$ . Note that large fusion hindrance for the tip-tip orientation of very heavy nuclei was found experimentally [21]. In contrast to that, the fusion hindrance is absent for the side-side orientation [21]. We see in Fig. 2 that deformation and mutual orientation of nuclei induce strong effect on the shape of the interaction potential. In contrast to this effect, the influence of deformation and mutual orientation for system  $^{70}\text{Zn} + ^{144}\text{Nd}$  is much smaller in Fig. 3, because deformation of  $^{70}\text{Zn}$  is small and nucleus  $^{144}\text{Nd}$  is spherical. Nevertheless comparing panels in the Fig. 3 we see 2 MeV variations of barrier heights induced by deformation of  $^{70}\text{Zn}$ . Nucleus  $^{144}\text{Nd}$  is spherical, therefore potentials corresponding to various values of  $\Theta_2$  are degenerated in Fig. 3.

Comparing results in Figs. 2 and 3 we conclude that the lowest barrier is taken place for the most elongated shape, while highest barrier is occurred for the most compact shape.

It is very interesting to study these reactions experimentally in the range 185 - 210 MeV for searching the fusion hindrance effect induced by deformation and mutual orientation of deformed nuclei. The elongated shapes associated with barriers

around 185 - 190 MeV may be related to the fusion hindrance, as it was observed for other similar systems [21].

The middle and right panels of Figs. 2 and 3 show the results evaluated for angle  $\Phi = 0^\circ$  and  $\Phi = 90^\circ$  respectively. We see in Figs. 2 and 3 that influence of orientation connected to the angle  $\Phi$  is very small.

The values of production cross-section of super-heavy elements (SHE) are very small [9, 10, 23, 24]. Therefore the nuclei participated in entrance-channel of fusion reactions and collision energies are very carefully selected in the case of reactions leading to SHE. Due to this the values of both the fusion barrier values and the excitation energies of compound-nucleus are especially important for the formation of SHE in nucleus-nucleus collisions [9, 13]. In Figs. 4 and 5 we discuss specific properties of the entrance-channel potential evaluated in the framework of our approach for reactions  $^{22}\text{Ne} + ^{248}\text{Cm}$  and  $^{26}\text{Mg} + ^{248}\text{Cm}$  respectively. The nuclei participated in both reactions are well-deformed  $\beta_2(^{22}\text{Ne}) = 0,326$ ,  $\beta_2(^{26}\text{Mg}) = -0,310$  and  $\beta_2(^{248}\text{Cm}) = 0,235$  [20, 22]. Therefore two prolate nuclei are interacting in the case of system  $^{22}\text{Ne} + ^{248}\text{Cm}$ , while oblate and prolate nuclei are interacting in collision of  $^{26}\text{Mg} + ^{248}\text{Cm}$ . Note that these systems are successfully used in experiments for the synthesis of Sg [23] and Hs [24].

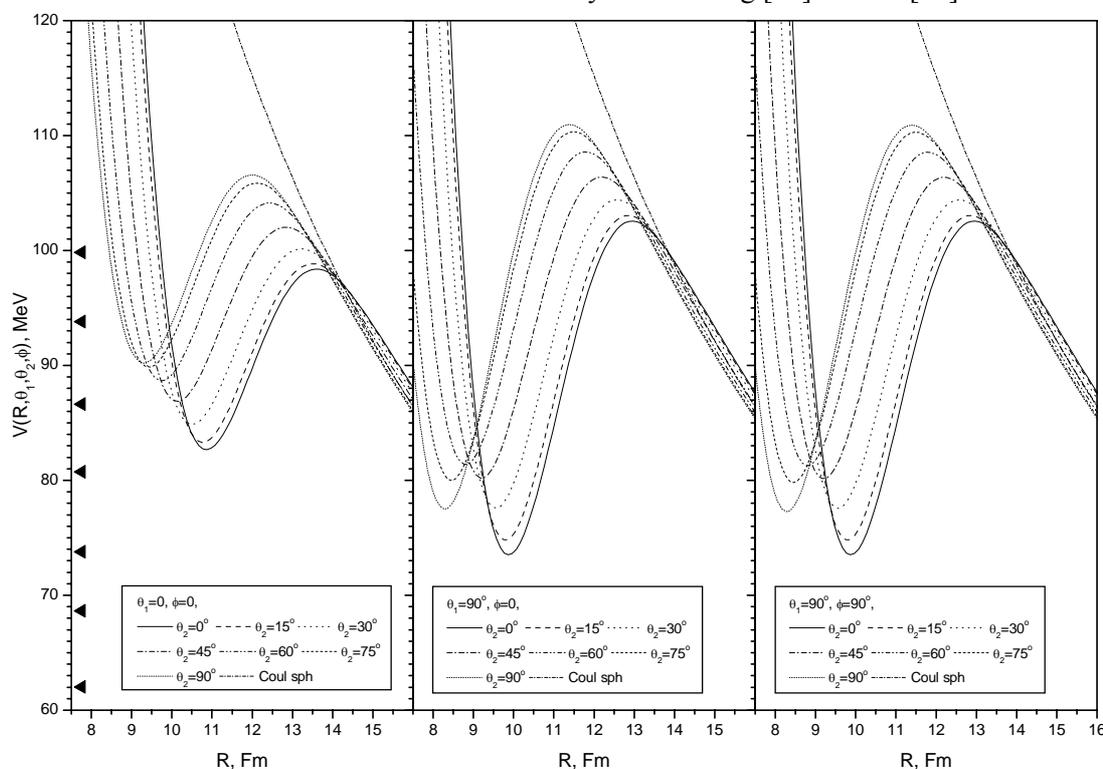


Fig. 4. The nucleus-nucleus potentials for the system  $^{22}\text{Ne} + ^{248}\text{Cm}$  for various orientations of both  $^{22}\text{Ne}$  and  $^{248}\text{Cm}$ . The Coulomb potential between these nuclei, when both nuclei are considered spherical, is also presented for comparison. The ground-state Q-values are indicated by the lowest triangles at the left vertical axes. The other 6 triangles mark, respectively, the thresholds for the emission of 1, 2, 3, 4, 5 and 6 neutrons.

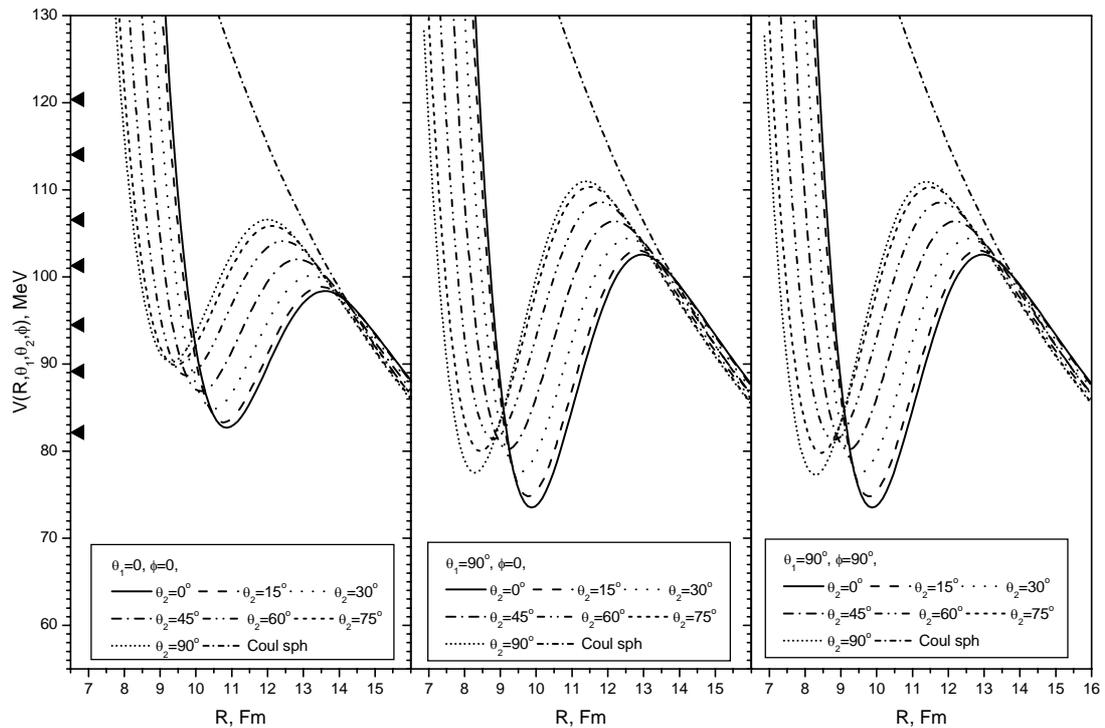


Fig. 5. The nucleus-nucleus potentials for the system  $^{26}\text{Mg} + ^{248}\text{Cm}$  for various orientations of both  $^{26}\text{Mg}$  and  $^{248}\text{Cm}$ . The notations are similar to the ones in Fig. 4.

The compact shapes corresponding to side-side orientations for prolate-prolate and prolate-oblate systems have highest values of the barrier, while the lowest values of barrier are taken place for the elongated tip-tip shape. So the orientation and deformation of heavy nucleus play leading role into the entrance-channel potential. Comparing middle and right panels in Figs. 4 and 5 we conclude that the orientation related to angle  $\Phi$  influence weakly on the potential.

There are many neutrons evaporating from compound-nuclei formed in the fusion reactions  $^{22}\text{Ne} + ^{248}\text{Cm}$  and  $^{26}\text{Mg} + ^{248}\text{Cm}$  [23, 24]. Therefore we present the ground-state reaction Q-values and the thresholds for the emission of 1, 2, 3, 4, 5 and 6 neutrons in Figs. 4 - 5. These quantities are obtained by using experimental binding energy [25].

Comparing the barriers of the potentials and the ground-state reaction Q-values for reactions in Figs. 4 and 5 we conclude that compound-nucleus formed in reaction  $^{26}\text{Mg} + ^{248}\text{Cm}$  at barrier energies is less excited than the one formed in reaction  $^{22}\text{Ne} + ^{248}\text{Cm}$  at barrier energies. So due to competition between neutron evaporation and fission of the compound-nucleus reaction  $^{26}\text{Mg} + ^{248}\text{Cm}$  is very attractive for the formation of the Hs for channels with 3 - 4 evaporated neutrons.

In conclusion we obtain the simple approach for the evaluation of the interaction potential between two deformed axial-symmetric nuclei. The main features of the potentials evaluated for various orientations of nuclei are discussed.

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## ПОТЕНЦІАЛ ВЗАЄМОДІЇ МІЖ ДВОМА АКСІАЛЬНО-СИМЕТРИЧНИМИ ЯДРАМИ

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Запропоновано простий підхід для визначення потенціалу взаємодії між двома аксіально-симетричними ядрами. Детально обговорюється потенціальна енергія деформованих, довільно орієнтованих, ядер. Розглядаються властивості потенціалу у вхідному каналі реакцій злиття  $^{22}\text{Ne} + ^{248}\text{Cm}$  та  $^{26}\text{Mg} + ^{248}\text{Cm}$  що приводять до утворення надважких елементів.

## ПОТЕНЦИАЛ ВЗАИМОДЕЙСТВИЯ МЕЖДУ ДВУМЯ АКСИАЛЬНО-СИММЕТРИЧНЫМИ ЯДРАМИ

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Предложен простой подход для определения потенциала взаимодействия между двумя аксиально-симметричными ядрами. Детально обсуждается потенциальная энергия деформированных, произвольно ориентированных, ядер. Рассматриваются свойства потенциала во входном канале реакций слияния  $^{22}\text{Ne} + ^{248}\text{Cm}$  и  $^{26}\text{Mg} + ^{248}\text{Cm}$ , приводящих к образованию сверхтяжелых элементов.

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