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NONLINEAR FIELD EFFECTS IN THE COLLECTOR RING DUE TO LARGE-AMPLITUDE PARTICLE OSCILLATIONS

In the frame of the FAIR project, the Collector Ring (CR) is planned to be built for efficient cooling of antiprotons and rare isotope beams [1]. In order to accept hot beam from separators large acceptances are required. This paper examines the effects which can influence the beam dynamics due to large betatron oscillation amplitude and momentum spread. Using analytic expressions, the amplitude-dependent tune shifts driven by sextupole magnets, the fringe field of quadrupole magnets, and kinematics effects have been calculated. The obtained results have been compared with numerical simulations by means of precise multi-turn particle tracing. The tracking analysis for the CR has been performed considering the real shape of the magnetic field of the wide aperture quadrupole. We report on quantitative studies of the effects on the dynamic aperture of the rings.

Keywords: collector ring, tune shift, chromaticity, kinematic effect, sextupole effect, fringe field.

1. Introduction

Tune spread in the CR has to be carefully evaluated as there are several effects that contribute to the variation of tune with amplitude [2]. This can cause a crossing of “dangerous” resonances and a subsequent dynamic aperture reduction. In this paper, we elaborate on the various sources of amplitude-dependent tune shift and list the contributions from each source to the particle motion. As an indication that tune spread is tolerable, we present how it influences the beam losses. Because of the large momentum spread (6 %) of an antiproton beam, the chromaticity of the CR is corrected by 24 sextupole magnets joint in 6 families. The sextupole magnets are the first source of amplitude-dependent tune shift. A second important source of amplitude-dependent tune shift is a fringe field of quadrupole magnets. The next source, which is significant for small storage rings, is kinematic effects that appear even for ideal magnets and alignment in the case of large beam sizes. The analytical estimation of the tune shift is based on the following expressions for the amplitude tune dependence [3, 4]

$$\begin{pmatrix} \Delta Q_x \\ \Delta Q_y \end{pmatrix} = \begin{pmatrix} C_{xx} & C_{xy} \\ C_{xy} & C_{yy} \end{pmatrix} \begin{pmatrix} J_x \\ J_y \end{pmatrix}, \tag{1}$$

where J_x and J_y are the horizontal and vertical action of the particle respectively. (Note: The two off-diagonal cross terms in Eq. (1) are equal.) The coefficients C include the contribution of the sextupole, kinematic and fringe field perturbations: $C = C^k + C^{sx} + C^{fr}$.

To prove the analytical calculations, numerical simulations have been performed. 15000 particles were tracked each with a different transverse starting position (x, x', y, y', $\Delta p/p$) for 1024 turns. We performed a FFT on the horizontal and vertical turn-by-turn position data and calculated the fractional horizontal and vertical tunes of the particles.

2. Sources of amplitude-dependent tune shift

2.1. Sextupole magnets

The first source of amplitude-dependent tune shift is 24 sextupole magnets, which are located in two CR arcs where the dispersion function is non-zero. Analytically the C^{sx} coefficients can be calculated by formulae [5, 6]

$$C_{xx}^{se} = -\frac{3}{128\pi\sin(\pi Q_x)} \oint\oint ds_1 ds_2 k_2(s_1) k_2(s_2) \cos\left(\left|\psi_x(s_1) - \psi_x(s_2) - \pi Q_x\right|\right) -$$

$$-\frac{1}{128\pi\sin(3\pi Q_x)} \oint\oint ds_1 ds_2 k_2(s_1) k_2(s_2) \cos\left(3\left|\psi_x(s_1) - \psi_x(s_2) - 3\pi Q_x\right|\right),$$

$$C_{yy}^{sx} = -\frac{3}{32\pi\sin(3\pi Q_x)}\oint\oint ds_1 ds_2 \vec{k}_2(s_1)\vec{k}_2(s_2)\cos\left(\left|\psi_x(s_1)-\psi_x(s_2)-\pi Q_x\right|\right)\mp$$

$$\mp\frac{1}{128\pi\sin(\pi Q_\pm)}\oint\oint ds_1 ds_2 \vec{k}_2(s_1)\vec{k}_2(s_2)\cos\left(\left|\psi_\pm(s_1)-\psi_\pm(s_2)-\pi Q_\pm\right|\right),$$

$$C_{xy}^{se} = \frac{1}{32\pi\sin(\pi Q_x)}\oint\oint ds_1 ds_2 k_2(s_1)\vec{k}_2(s_2)\cos\left(\left|\psi_x(s_1)-\psi_x(s_2)-\pi Q_x\right|\right)-$$

$$-\frac{1}{64\pi\sin(\pi Q_\pm)}\oint\oint ds_1 ds_2 \vec{k}_2(s_1)\vec{k}_2(s_2)\cos\left(\left|\psi_\pm(s_1)-\psi_\pm(s_2)-\pi Q_\pm\right|\right) \quad (2)$$

ψ_x, ψ_y are the betatron phase angles (0 to $2\pi Q_{xy}$). $\psi_\pm = 2\psi_y \pm \psi_x$, $Q_\pm = 2Q_y \pm Q_x$ and $Q_{x,y}$ are the horizontal and vertical betatron tunes.

$$k_2(s) = \beta_x^{3/2}(s)\frac{1}{BR}\frac{\partial^2 B}{\partial x^2},$$

$$\vec{k}_2(s) = \sqrt{\beta_x(s)}\beta_y(s)\frac{1}{BR}\frac{\partial^2 B}{\partial x^2},$$

where BR is the magnet rigidity and $\beta_{x,y}$ are the horizontal and vertical betatron functions. For the CR the betatron functions are shown in Fig. 1.

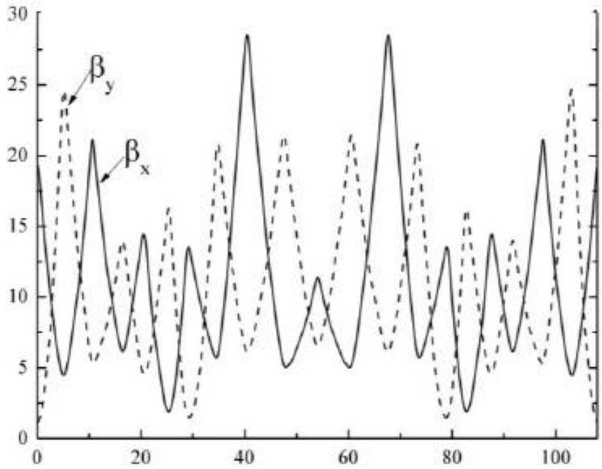

Fig. 1. Betatron functions over half of the CR ring.

In order to compare the tune shifts calculated by the analytical expressions with numerical simulation, the latter was carried out for the case of thin lens approximation of the sextupole magnets, where the particles angles are changed by

$$\Delta x' = m\left[(x+D\delta)^2 - y^2\right]\cdot 0.5,$$

$$\Delta y' = -m(x+D\delta)y. \quad (3)$$

where $m = (L/BR)d^2B/dx^2$ is the integrated strength of a sextupole magnet, $\delta = \Delta p/p$, D is the horizontal dispersion function.

2.2. Kinematic effect

The kinematic non-linearity arises from high-order terms proportional to the transverse momenta p_x, p_y into the expansion of the standard square-root relativistic Hamiltonian. The first correction to the tune shift comes from octupole-like terms: p_x^4, $p_x^2p_y^2$ and p_y^4. A transformation to action variables results in the kinematic coefficients [4]

$$C_{xx}^k = \frac{3}{16\pi}\oint\gamma_x^2(s)ds, \quad C_{yy}^k = \frac{3}{16\pi}\oint\gamma_y^2(s)ds,$$

$$C_{xy}^k = \frac{1}{8\pi}\oint\gamma_x\gamma_y(s)ds, \quad (4)$$

where γ is the characteristic of the CR (one of the Twiss parameters). For the CR these functions are relatively large as shown in Fig. 2.

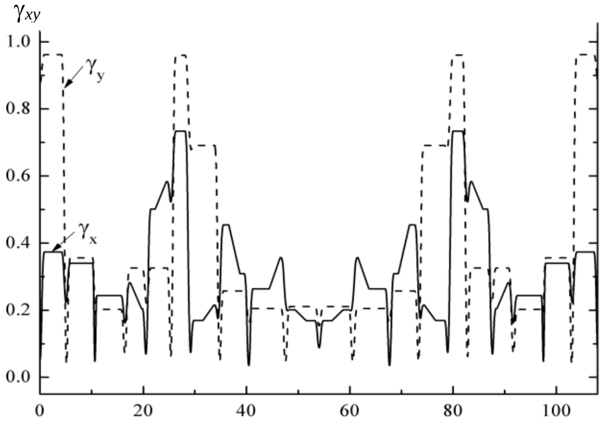

Fig. 2. The horizontal and vertical $\gamma_{x,y}$ functions of the CR.

The beam emittances (actions) at the CR are also large, thus both effects produce noticeable kinematic tune shifts. Such kinematic tune shifts were observed in our numerical simulations.

2.3. Fringe field

The fringe field is another source of large amplitude contribution to the tune shifts. In the CR, the normal-conducting quadrupoles in the arcs have a horizontal aperture of 40 cm, which leads to strong, extended fringe fields proportional to the pole tip field of the quadrupole up to 1 T. With the need to minimize arc lengths in order to have good properties for stochastic cooling, the pole tip field and the fringe fields cannot be reduced by correspondingly lengthening the quadrupoles. Hence the impact of the quadrupole end fields on the performance of this CR must be carefully evaluated. Therefore, the contribution from quadrupole magnet ends becomes significant. Analytically the fringe field coefficients in Eq. (1) can be calculated by expressions [4]:

$$C_{xx}^{fr} = -\frac{1}{32\pi}\oint k_1''(s)\beta_x^2(s)ds,$$

$$C_{yy}^{fr} = +\frac{1}{32\pi}\oint k_1''(s)\beta_y^2(s)ds,$$

$$C_{xy}^{fr} = -\frac{1}{16\pi}\oint \beta_x(s)\left(k_1''(s)\beta_y(s) - 4k_1'\alpha_y(s)\right)ds. \quad (5)$$

Here *k, k', k''* are quadrupole, sextupole, and octupole strengths. The shape of the quadrupole fringe field and its derivatives have been calculated and presented in Fig. 3 in relative values [7]. The maximum values are k = 0.274 m^{-2}, $k'_{\max}$ = 0.866 m^{-3}, $k''_{\max}$ = 5.325 m^{-4} [2].

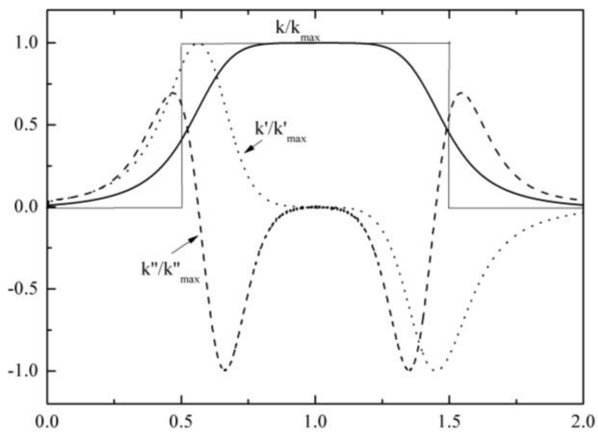

Fig. 3. Relative values of the CR quadrupole fringe field and its derivatives.

3. Tune shift from various sources

The tune shift due to quadrupole fringe fields, kinematic effect, and sextupole magnets in the CR is investigated by means of multiturn ray tracing and compared with analytical simulations based on Eq. (1). We consider only ion-optical settings in the antiproton mode operation. In the following, we first establish the basic optical properties, which are resulted in the betatron functions (see Fig. 1). For larger amplitude oscillations the particles include momentum deviation up to ±3 %. The chromaticity is corrected in the absence of fringe fields by means of the 6 sextupole families. Next, quadrupole fringe fields are introduced, and tune shift is calculated. The kinematic effect is calculated with the absence of sextupole magnets and fringe fields. The quantitative results of the tune shift calculated by analytical and numerical simulations are given in Table.

Tune shift in the CR from various sources

Magnetic elements	C_{xx}	C_{yy}	C_{xy}
Sextupole			
analytic	1.525	–4.304	–0.238
numerical	1.212	–1.712	–0.851
Quantitative fringe			
analytic	16.222	20.551	16.122
numerical	20.952	12.254	–1.213
Kinematics			
analytic	1.480	1.991	1.177
numerical	2.211	1.354	0.658

It is shown that the fringe field of the quadrupole magnets has the main contribution to the tune shift. It is larger than that of the regular sextupole magnets used for chromatics corrections. The impact of the kinematic effects on the tune shift is also large but lower than from the fringe field. For the CR one can point out that the dynamical aperture is limited mostly by the kinematic terms of some of the drift space and by the fringe field of the quadrupoles in the arcs.

In Fig. 4 we show the tune shifts for the CR ring produced solely by the kinematic corrections (no non-linear elements in the lattice or magnet errors are present). This plot is generated by launching particles in five different directions with the transverse amplitudes going up to 240 mm·mrad.

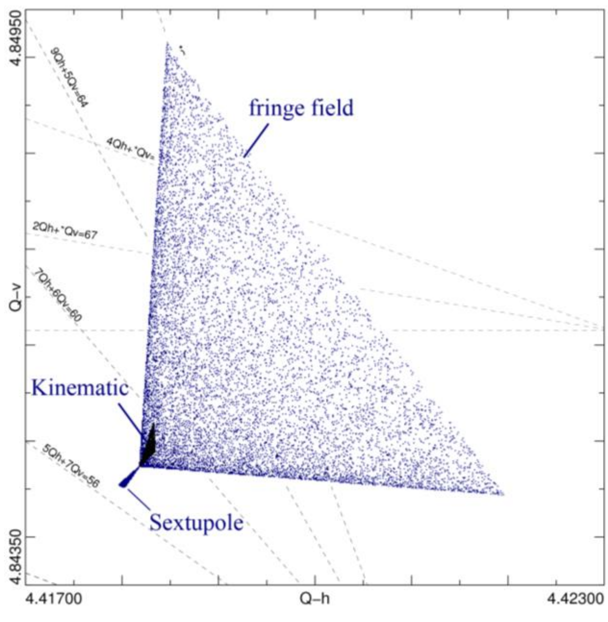

Fig. 4. Tune shift versus particle amplitude in the CR.

In Fig. 4 the calculated tune shift by means of the particle tracking for the various sources is shown. One can see the expected beam population near the tune plane area to the right-up of the working point which is $Q_x = 4.4182$, $Q_y = 4.8445$.

As an illustration of this we tracked about 10^5 particles each with a different transverse starting position, (Q_x, Q_y), for 1024 turns and if the particle survived 1024 turns, we performed a FFT on the horizontal and vertical turn-by-turn position data and calculated the particle's fractional horizontal and vertical tunes. Fig. 4 shows 10^5 points, each corresponding to one particle launched on a grid inside the dynamic aperture of the CR. We see that both the horizontal and vertical betatron tunes are dominantly dependent on the fringing field effect.

4. Conclusions

In this paper, we present the effects, which influence the particle motion in the CR storage ring. Three sources are considered: the quadrupole fringe field, kinematic, and sextupole. Each effect is calculated individually and independently from the other. We analyze the betatron tune shifts as a function of the particle amplitude. These simulations allow us to identify a dominant effect if there is no compensation. As a consequence, the expected particle loss can be defined, but this is not a subject of this paper.

The effects on various machine parameters and on transverse and momentum acceptance in the CR have been investigated by means of multiturn ray tracing. It was shown that in the antiproton mode of the CR operation for given betatron tunes the quadrupole fringing field is dominated over the other effects.

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ЕФЕКТИ НЕЛІНІЙНОГО ПОЛЯ В НАКОПИЧУВАЛЬНОМУ ПРИСКОРЮВАЧІ ЧЕРЕЗ ВЕЛИКОАМПЛІТУДНІ ОСЦИЛЯЦІЇ ЧАСТИНОК

У рамках проекту FAIR заплановано побудувати накопичувальне кільце (Collector Ring – CR) з метою ефективного охолодження пучків антипротонів і рідкісних ізотопів [1]. Оскільки після сепарації потрібно приймати гарячі пучки, необхідно забезпечити великий аксептанс на приймаючому прискорювачі. У статті розглядаються ефекти, що можуть впливати на динаміку пучка через велику амплітуду бетатронних коливань та розкид імпульсу. Використовуючи аналітичні вирази, було розраховано залежні від амплітуди зміщення частоти коливань частинок, викликані впливом секступольних магнітів, краєвим полем квадрупольних магнітів та кінематичними ефектами. Отримані результати порівнюються з чисельним моделюванням за допомогою прецизійного дослідження мультитрекінгу частинок на прискорювачі CR, що проводилися з урахуванням реальної форми магнітного поля широкоапертурного квадруполя. Надано результати чисельного дослідження впливу зазначених ефектів на динамічну апертуру кільцевих прискорювачів.

Ключові слова: накопичувальне кільце, зміщення частоти, хроматичність, кінематичний ефект, секступольний ефект, краєве поле.

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