

**А. Х. Такі\*, Г. А. Мохаммед**

*Факультет фізики, Науковий коледж, Кіркукський університет, Кіркук, Ірак*

\*Відповідальний автор: alitaqi@uokirkuk.edu.iq

## **ІЗОСКАЛЯРНИЙ МОНОПОЛЬНИЙ ВІДГУК У НЕЙТРОННО-БАГАТИХ ІЗОТОПАХ МОЛІБДЕНУ З ВИКОРИСТАННЯМ САМОУЗГОДЖЕНОГО НАБЛИЖЕННЯ QRPA**

Ізоскалярний гігантський монопольний резонанс (ISGMR) парних ізотопів молібдену  $^{92,94,96,98,100}\text{Mo}$  вивчався в рамках самоузгодженого наближення Хартрі - Фока - Бардіна, Купера та Шріффера і квазічастинкового наближення випадкових фаз. У розрахунках використано десять наборів взаємодій типу Скірма з різними значеннями коефіцієнта нестисливості ядерної матерії  $K_{\text{NM}}$ . Розраховані розподіли сил, центродійних енергій  $E_{\text{cen}}$ , ренормованих енергій  $E_s$  і обмежених енергій  $E_{\text{con}}$  в ISGMR порівнюються з наявними експериментальними даними. При відповідному значенні нестисливості ядерної матерії  $K_{\text{NM}}$  кілька типів взаємодій Скірма були успішними в описі розподілу сил ISGMR в ізотопах  $^{92,94,96,98,100}\text{Mo}$ . У результаті було виявлено великі кореляції між  $E_{\text{cen}}$  і  $K_{\text{NM}}$ .

*Ключові слова:* розподіл сил, сили Скірма, наближення Хартрі - Фока - Бардіна - Купера - Шріффера, квазічастинкове наближення випадкових фаз.

**A. H. Taqi\*, G. A. Mohammed**

*Department of Physics, College of Science, Kirkuk University, Kirkuk, Iraq*

\*Corresponding author: alitaqi@uokirkuk.edu.iq

## **ISOSCALAR MONOPOLE RESPONSE IN THE NEUTRON-RICH MOLYBDENUM ISOTOPES USING SELF-CONSISTENT QRPA**

The isoscalar giant monopole resonance (ISGMR) of even molybdenum isotopes  $^{92,94,96,98,100}\text{Mo}$  has been studied within the Skyrme self-consistent Hartree - Fock - Bardeen, Cooper, and Schrieffer and quasi-particle random phase approximation. Ten sets of Skyrme-type interactions of different values of the nuclear matter incompressibility coefficient  $K_{\text{NM}}$  are used in the calculations. The calculated strength distributions, centroid energies  $E_{\text{cen}}$ , scaled energies  $E_s$  and constrained energies  $E_{\text{con}}$  of ISGMR are compared with available experimental data. Due to the appropriate value of the nuclear matter incompressibility  $K_{\text{NM}}$ , several types of Skyrme interactions were successful in describing the ISGMR strength distribution in the  $^{92,94,96,98,100}\text{Mo}$  isotopes. As a result, high correlations between  $E_{\text{cen}}$  and  $K_{\text{NM}}$  were found.

*Keywords:* strength distribution, Skyrme force, Hartree - Fock - Bardeen - Cooper - Schrieffer, quasiparticle random phase approximation.

## **REFERENCES**

1. N.V. Giai. Self-Consistent Description of Nuclear Excitations. *Prog. Theor. Phys. Supp.* **74-75** (1983) 330.
2. E. Khan, N.V. Giai. Low-lying  $2^+$  states in neutron-rich oxygen isotopes in quasiparticle random phase approximation. *Phys. Lett. B* **472** (2000) 253.
3. E. Khan et al. Low-lying collective states in neutron-rich oxygen isotopes via proton scattering. *Phys. Lett. B* **490** (2000) 45.
4. E. Khan et al. Proton scattering from the unstable nuclei  $^{30}\text{S}$  and  $^{34}\text{Ar}$ : structural evolution along the sulfur and argon isotopic chains. *Nucl. Phys. A* **694** (2001) 103.
5. J. Dobaczewski et al. Mean-field description of ground-state properties of drip-line nuclei: Pairing and continuum effects. *Phys. Rev. C* **53** (1996) 2809.
6. J. Engel et al.  $\beta$  decay rates of r-process waiting-point nuclei in a self-consistent approach. *Phys. Rev. C* **60** (1999) 014302.
7. M. Matsuo. Continuum Linear Response in Coordinate Space Hartree-Fock-Bogoliubov Formalism for Collective Excitations in Drip-line Nuclei. *Nucl. Phys. A* **696** (2001) 371.
8. E. Khan et al. Detecting bubbles in exotic nuclei. *Nucl. Phys. A* **800** (2008) 37.
9. O. Sorlin, M.-G. Porquet. Nuclear magic numbers: new features far from stability. *Prog. Part. Nucl. Phys.* **61** (2008) 602.
10. W.A. Mansour, A.H. Taqi. Isoscalar Giant Dipole Resonance of Tin Isotopes  $^{112, 114, 116, 118, 120, 122, 124}\text{Sn}$  Using HF-BCS and QRPA Approximations. *Kirkuk J. Sci.* **18(4)** (2023) 42.
11. T. Li et al. Isotopic dependence of the giant monopole resonance in the even-A  $^{112-124}\text{Sn}$  isotopes and the

- asymmetry term in nuclear incompressibility. *Phys. Rev. Lett.* **99** (2007) 162503.
12. C. Monrozeau et al. First Measurement of the Giant Monopole and Quadrupole Resonances in a Short-Lived Nucleus:  $^{56}\text{Ni}$ . *Phys. Rev. Lett.* **100** (2008) 042501.
  13. T. Li et al. Isoscalar giant resonances in the Sn nuclei and implications for the asymmetry term in the nuclear-matter incompressibility. *Phys. Rev. C* **81** (2010) 034309.
  14. D. Patel et al. Giant monopole resonance in even-A Cd isotopes, the asymmetry term in nuclear incompressibility, and the “softness” of Sn and Cd nuclei. *Phys. Lett. B* **718** (2012) 447.
  15. J. Piekarewicz. Why is the equation of state for tin so soft? *Phys. Rev. C* **76** (2007) 031301(R).
  16. J. Li, G. Colò, J. Meng. Microscopic linear response calculations based on the Skyrme functional plus the pairing contribution. *Phys. Rev. C* **78** (2008) 064304.
  17. E. Khan. Role of superfluidity in nuclear incompressibilities. *Phys. Rev. C* **80** (2009) 011307(R).
  18. P. Veselý et al. Giant monopole resonances and nuclear incompressibilities studied for the zero-range and separable pairing interactions. *Phys. Rev. C* **86** (2012) 024303.
  19. E. Khan, J. Margueron, I. Vidaña, Constraining the Nuclear Equation of State at Subsaturation Densities. *Phys. Rev. Lett.* **109** (2012) 092501.
  20. L.-G. Cao, H. Sagawa, G. Colò. Microscopic study of the isoscalar giant monopole resonance in Cd, Sn, and Pb isotopes. *Phys. Rev. C* **86** (2012) 054313.
  21. L. Capelli, G. Colò, J. Li. Dielectric theorem within the Hartree-Fock-Bogoliubov framework. *Phys. Rev. C* **79** (2009) 054329.
  22. E. Khan, N. Paar, D. Vretenar. Low-energy monopole strength in exotic nickel isotopes. *Phys. Rev. C* **84** (2011) 051301(R).
  23. E. Khan et al. Incompressibility of finite fermionic systems: Stable and exotic atomic nuclei. *Phys. Rev. C* **87** (2013) 064311.
  24. E. Yüksel, E. Khan, K. Bozkurt. The soft Giant Monopole Resonance as a probe of the spin-orbit splitting. *Eur. Phys. J. A* **49** (2013) 124.
  25. J.P. Blaizot. Nuclear compressibilities. *Phys. Rep.* **64** (1980) 171.
  26. J.P. Blaizot et al. Microscopic and macroscopic determinations of nuclear compressibility. *Nucl. Phys. A* **591** (1995) 435.
  27. G.A. Mohammed, A.H. Taqi. Isoscalar Dipole Response in  $^{92}\text{Mo}$  and  $^{100}\text{Mo}$  Isotopes. *Momento* **67** (2023) 101.
  28. S. Shlomo, V.M. Kolomietz, G. Colò. Deducing the nuclear-matter incompressibility coefficient from data on isoscalar compression modes. *Eur. Phys. J. A* **30** (2006) 23.
  29. D.H. Youngblood, H.L. Clark, Y.-W. Lui. Incompressibility of Nuclear Matter from the Giant Monopole Resonance. *Phys. Rev. Lett.* **82** (1999) 691.
  30. S. Shlomo, D.H. Youngblood. Nuclear matter compressibility from isoscalar giant monopole resonance. *Phys. Rev. C* **47** (1993) 529.
  31. S.H. Amin, A.A. Al-Rubaiee, A.H. Taqi. Effect of Incompressibility and Symmetry Energy Density on Charge Distribution and Radii of Closed-Shell Nuclei. *Kirkuk Journal of Science* **17**(3) (2022) 17.
  32. D.R. Lide (Ed.). *CRC Handbook of Chemistry and Physics*. 87th ed. (CRC Press, 2006) 2608 p.
  33. A. Moalem et al. Isotopic dependence of the giant quadrupole resonance in the stable even-mass molybdenum nuclei. *Phys. Rev. C* **20** (1979) 1593(R).
  34. G. Duhamel et al. Inelastic alpha scattering to the giant quadrupole and monopole resonances of  $^{58}\text{Ni}$ ,  $^{92}\text{Mo}$ , and  $^{120}\text{Sn}$  at 152 MeV. *Phys. Rev. C* **38** (1988) 2509.
  35. D.H. Youngblood et al. Unexpected characteristics of the isoscalar monopole resonance in the  $A \approx 90$  region: Implications for nuclear incompressibility. *Phys. Rev. C* **88** (2013) 021301(R).
  36. D.H. Youngblood et al. Isoscalar E0, E1, E2, and E3 strength in  $^{92,96,98,100}\text{Mo}$ . *Phys. Rev. C* **92** (2015) 014318.
  37. Krishchayan et al. Isoscalar giant resonances in  $^{90,92,94}\text{Zr}$ . *Phys. Rev. C* **92** (2015) 044323.
  38. K.B. Howard et al. Compressional-mode resonances in the molybdenum isotopes: Emergence of softness in open-shell nuclei near  $A = 90$ . *Phys. Lett. B* **807** (2020) 135608.
  39. G. Colò et al. Isoscalar monopole and quadrupole modes in Mo isotopes: Microscopic analysis. *Phys. Lett. B* **811** (2020) 135940.
  40. B.K. Agrawal, S. Shlomo, V. Kim Au. Determination of the parameters of a Skyrme type effective interaction using the simulated annealing approach. *Phys. Rev. C* **72** (2005) 014310.
  41. Z. Zhfng, L.-W. Chen. Extended Skyrme interactions for nuclear matter, finite nuclei, and neutron stars. *Phys. Rev. C* **94** (2016) 064326.
  42. B.A. Brown. New Skyrme interaction for normal and exotic nuclei. *Phys. Rev. C* **58** (1998) 220.
  43. Q. Shen, Y. Han, H. Guo. Isospin dependent nucleon nucleus optical potential with Skyrme interactions. *Phys. Rev. C* **80** (2009) 024604.
  44. J.M. Pearson, S. Goriely. Isovector effective mass in the Skyrme-Hartree-Fock method. *Phys. Rev. C* **64** (2001) 027301.
  45. P.-G. Reinhard et al. Shape coexistence and the effective nucleon-nucleon interaction. *Phys. Rev. C* **60** (1999) 014316.
  46. B.A. Brown et al. Neutron skin deduced from antiprotonic atom data. *Phys. Rev. C* **76** (2007) 034305.

47. H.S. Köhler. Skyrme force and the mass formula. *Nucl. Phys. A* **258** (1976) 301.
48. S. Krewald et al. On the use of Skyrme force in self-consistent RPA calculations. *Nucl. Phys. A* **281** (1977) 166.
49. T.H.R. Skyrme. The effective nuclear potential. *Nucl. Phys. 9 (1958-1959)* 615.
50. A.H. Taqi, M.S. Ali. Self-consistent Hartree-Fock RPA calculations in  $^{208}\text{Pb}$ . *Indian J. Phys. 92(1)* (2018) 69.
51. J.R. Stone, P.-G. Reinhard. The Skyrme interaction in finite nuclei and nuclear matter. *Prog. Part. Nucl. Phys.* **58(2)** (2007) 587.
52. D. Vautherin, D.M. Brink. Hartree-Fock calculations with Skyrme's interaction. I. Spherical Nuclei. *Phys. Rev. C 5 (1972)* 626.
53. M. Bender, P.-H. Heenen, P.-G. Reinhard. Self-consistent mean-field models for nuclear structure. *Rev. Mod. Phys. 75 (2003)* 121.
54. P. Ring, P. Schuck. *The Nuclear Many-Body Problem* (Heidelberg, Springer Berlin, 1980) 718 p.
55. E. Chabanat et al. A Skyrme parametrization from subnuclear to neutron star densities Part II. Nuclei far from stabilities. *Nucl. Phys. A 635 (1998)* 231.
56. W. Ryssens et al. Solution of the Skyrme-HF+BCS equation on a 3D mesh, II: A new version of the Ev8 code. *Computer Physics Communications 187 (2015)* 175.
57. D.J. Rowe. *Nuclear Collective Motion: Models and Theory* (London, Methuen, 1970) 340 p.
58. G. Colò et al. Self-consistent RPA calculations with Skyrme-type interactions: The *skyrme\_rpa* program. *Computer Physics Communications 184 (2013)* 142.
59. A.H. Taqi, G.L. Alawi. Isoscalar giant resonance in  $^{100,116,132}\text{Sn}$  isotopes using Skyrme HF-RPA. *Nucl. Phys. A 983 (2019)* 103.
60. A.H. Taqi, E.G. Khidher. Ground and transition properties of  $^{40}\text{Ca}$  and  $^{48}\text{Ca}$  nuclei. *Nucl. Phys. At. Energy 19 (2018)* 326.
61. S. Stringari. Sum rules for compression modes. *Phys. Lett. B 108* (1982) 232.
62. J. Button et al. Isoscalar E0, E1, E2, and E3 strength in  $^{94}\text{Mo}$ . *Phys. Rev. C 94 (2016)* 034315.
63. M. Itoh et al. Systematic study of  $L \leq 3$  giant resonances in Sm isotopes via multipole decomposition analysis. *Phys. Rev. C 68 (2003)* 064602.
64. U. Garg et al. Splitting of the giant monopole resonance with deformation in Sm nuclei. *Phys. Rev. Lett. 45 (1980)* 1670.
65. S. Brandenburg et al. Fission decay of the isoscalar giant monopole resonance in  $^{238}\text{U}$ . *Phys. Rev. Lett. 49 (1982)* 1687.
66. Y.K. Gupta et al. Splitting of ISGMR strength in the light-mass nucleus  $^{24}\text{Mg}$  due to ground-state deformation. *Phys. Lett. B 748 (2015)* 343.
67. Y.K. Gupta et al. Deformation effects on isoscalar giant resonances in  $^{24}\text{Mg}$ . *Phys. Rev. C 93 (2016)* 044324.
68. T. Peach et al. Effect of ground-state deformation on isoscalar giant resonances in  $^{28}\text{Si}$ . *Phys. Rev. C 93 (2016)* 064325.
69. Y.K. Gupta et al. Isoscalar giant monopole, dipole, and quadrupole resonances in  $^{90,92}\text{Zr}$  and  $^{92}\text{Mo}$ . *Phys. Rev. C 97 (2018)* 064323.
70. Y.K. Gupta et al. Are there nuclear structure effects on the isoscalar giant monopole resonance and nuclear incompressibility near  $A \sim 90$ ? *Phys. Lett. B 760 (2016)* 482.
71. M.N. Harakeh, A. van der Woude. *Giant Resonances: Fundamental High-Frequency Modes of Nuclear Excitation* (Oxford University Press, New York, 2001) 638 p.
72. B.K. Jennings, A.D. Jackson. Sum rules and the breathing mode. *Nucl. Phys. A 342 (1980)* 23.

Надійшла/Received 05.05.2023