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STUDY ON THE DECAY OF Z = 127 – 132 SUPERHEAVY NUCLEI VIA EMISSION OF 1-N AND 2-N HALO NUCLEI

The barrier penetrability, decay constant and decay half-life of 1-n halo nuclei ¹¹Be, ^{15,17,19}C, ²²N, ²³O, ^{24,26}F, ^{29,31}Ne, ^{34,37}Na, ^{35,37}Mg, and ⁵⁵Ca; and 2-n halo nuclei ²²C, ^{27,29}F, ³⁴Ne, ³⁶Na, and ⁴⁶P from Z = 127 - 132 parents were calculated within the framework of the Coulomb and proximity potential model by calculating the Q-values using the finite-range droplet model. A comparison between the decay half-lives is made by considering the halo candidates as a normal cluster and as a deformed structure with a rms radius. Neutron shell closure at 190, 196, 198, 200, 204, and 208 are identified from the plot of decay half-lives versus the neutron number of daughter nuclei (*N_P*). The calculation of alpha decay half-life and spontaneous decay half-life showed that the majority of the parent nuclei survive spontaneous fission and decay through alpha emission. The Geiger - Nuttall plots of $\log_{10} T_{1/2}$ versus $Q^{-1/2}$ and universal plots of $\log_{10} T_{1/2}$ versus $-\ln P$ for the emission of all 1-n and 2-n halo nuclei from the parents considered here are linear and show the validity of Geiger - Nuttall law in the case of decay of halo nuclei from superheavy elements.

Keywords: cluster radioactivity, halo nuclei, superheavy elements.

1. Introduction

A halo is a weakly bound nuclear system in which the last one or two nucleons remain beyond the interaction potential of the nucleus and hence possess a much larger matter radius than a normal nucleus [1, 2]. For such nucleons, the separation energy is extremely small, usually of the order of 1 MeV or even less. The halo can be either a neutron halo or a proton halo [3, 4]. The existence of a halo was first identified in ¹¹Li nuclei in 1985 by Tanihata et al [5]. The existence of a halo in ¹¹Li, ¹¹Be, ¹⁴Be, ¹⁴B, ¹⁵C, and ¹⁹C was experimentally confirmed in the laboratory [6]. Other proposed candidates for neutron halo include ^{6,8}He, ¹²Be, ^{17,19}B, ^{17,22}C, ²²N, ²³O, ^{24,26,27,29}F, and ²⁹Ne [2, 7]. The first experimentally produced neutron halo nucleus is ⁶He [8]. After the introduction of radioactive beam facilities, studies on the structure and other properties of halo nuclei became more popular among nuclear scientists [9 - 13].

The shell model and mean field approaches cannot be used successfully for describing the halo structure of a nucleus [14]. The reaction cross-section measurements are widely used to obtain information about the existence and structure of halo nuclei [15 - 18]. In 2009, by measuring the reaction cross-sections for single-neutron removal from the very neutron-rich nucleus ³¹Ne on lead and carbon targets at an energy of 230 MeV per nucleon using

the RIBF facility at RIKEN, T. Nakamura et al. predicted the halo structure of ³¹Ne nucleus as s- or p-orbital halo [19]. In 2014, Takechi et. al. confirmed the existence of a ³⁷Mg halo nucleus through the measurement of reaction cross-section [20].

Apart from the large matter radius, halo nuclei exhibit new magic numbers resulting from the dynamic effects of nucleon-nucleon interaction. The emergence of new magic numbers can be predicted through the study of the stability of parent nuclei against alpha decay and by using relativistic meanfield calculations [21, 22]. To study the structure and formation of halo nuclei, many structural models that include two-body systems, three-body systems, and microscopic models as well as many reaction models are available [23 - 26].

In the present work, our aim is to study the possibility of the emission of 1- and 2-neutron halo nuclei from parents with Z = 127 - 132, in the superheavy region. The studies on the synthesis of superheavy elements became more popular after the establishment of experimental facilities for hot fusion reactions at JINR-FLNR (Russia) and at GSI (Germany), and cold fusion reactions at RIKEN (Japan) [27 - 32]. Superheavy elements with Z = 112 - 118 were successfully synthesized through various fusion reactions with a ⁴⁸Ca beam on various actinide targets at FLNR (Dubna), GSI (Darmstadt), and LBNL (Berkeley) [33]. The last element that was

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successfully synthesized is Oganesson (Z = 118, A = 294) through the 3n-evaporation channel of the ²⁴⁹Cf + ⁴⁸Ca reaction [34]. Recently, many theoretical and experimental studies have been conducted on the synthesis of Z = 119 and 120 nuclei [35 - 38]. In 2014, Zhu et al., studied the reaction cross-sections based on ${}^{48}Ca + {}^{252}Es$, ${}^{50}Ti + {}^{249}Bk$, and ${}^{51}V + {}^{248}Cm$ fusion reactions for the synthesis of Z = 119 and 120 nuclei [39]. Many experimental attempts were also made to synthesize these elements [33, 40, 41]. The experimental studies on the synthesis of elements with Z = 119 and higher through hot fusion reactions require highly intense beams and improved detection facilities. In 2016, Santhosh et al. studied the different decay modes of Z = 104 - 136 even-even nuclei [42]. Our study is motivated by the synthesis of superheavy elements by fusion reactions and the prediction of the existence of elements with higher atomic numbers such as 124, 126, and 128 [43 - 45]. We would like to point out that we have already made an attempt to study the formation of proton halo nuclei from superheavy elements and the results were published [46, 47].

In Section 2, the details of the model used for the study are discussed. In Section 3, the results of our calculations are given.

2. The Coulomb and proximity potential model

For our study, we have used the Coulomb and proximity potential model (CPPM), which is one of the most successful models used to describe the decay of heavy fragments from elements in heavy and superheavy regions. This model assumes the interaction potential barrier as the sum of Coulomb, proximity, and centrifugal potentials for the touching configuration and for the separated fragments [48]. For the overlap region, a simple power law interpolation provided by Shi and Swiatecki [49] is used. The introduction of proximity potential makes the barrier more realistic and reduces the height of the barrier. Also, the results of calculations obtained through this model are closely in agreement with the experimental data. For a parent nucleus exhibiting exotic decay, the interacting potential barrier can be written as

$$V = \frac{Z_1 Z_2 e^2}{r} + V_p(z) + \frac{\hbar^2 l(l+1)}{2\mu r^2}; \text{ for } z > 0.$$
(1)

In the above expression, Z_1 and Z_2 are the atomic numbers of the daughter and the emitted cluster, z is the distance between the near surface of the fragments, r is the distance between the fragment centres, which is given as $r = z + C_1 + C_2$. The quantity l is the angular momentum quantum number, μ is the reduced mass and $V_p(z)$ is the proximity potential. The proximity potential is given by Blocki et al. [50] as

$$V_{p}(z) = 4\pi\gamma b \left[\frac{C_{1}C_{2}}{(C_{1}+C_{2})}\right] \Phi\left(\frac{z}{b}\right), \qquad (2)$$

where γ is the nuclear surface tension coefficient, *b* is the width of the nuclear surface (diffuseness), *C_i* are Süssmann central radii and Φ the universal proximity potential. These equations are applied to spherical nuclei.

The nuclear surface tension coefficient [48] is given by

$$\gamma = 0.9517 \Big[1 - 1.7826 (N - Z)^2 / A^2 \Big] \text{ MeV/fm}^2, (3)$$

where N, Z and A represents the neutron number, proton number and mass number of the nucleus. The universal proximity potential [50] is given by the expression

$$\Phi(\varepsilon) = -4.41e^{-\varepsilon/0.7176}, \text{ for } \varepsilon \ge 1.9475 \qquad (4)$$

and

 $\Phi(\varepsilon) = -1.7817 + 0.9270\varepsilon + 0.0169\varepsilon^2 - 0.05148\varepsilon^3,$

for
$$0 \le \varepsilon \le 1.9475$$
, (5)

where $\varepsilon = (z/b)$ with $b \approx 1$. The Süssmann central radii C_i of fragments are related to the sharp radii R_i as

$$C_i = R_i - \left(\frac{b^2}{R_i}\right). \tag{6}$$

The sharp radii R_i can be calculated by using an empirical formula in terms of the mass numbers A_i as [51]

$$R_i = 1.28A_i^{1/3} - 0.76 + 0.8A_i^{-1/3}.$$
 (7)

The potential for the overlap region of the barrier is given as

$$V = a_0 (L - L_0)^n$$
, for $z < 0.$ (8)

In this expression, $L = z + 2C_1 + 2C_2$ and $L_0 = 2C$. The constant a_0 and the parameter *n* are determined by the smooth matching of the two potentials at the touching point.

The barrier penetrability P can be obtained by using one-dimensional WKB approximation and is

given as

$$P = \exp\left\{-\frac{2}{\hbar}\int_{a}^{b}\sqrt{2\mu(V-Q)}dz\right\},$$
 (9)

where a and b are the turning points given by V(a) = V(b) = Q and Q is the energy released in the decay process.

Also $\mu = mA_1A_2 / A$ is the reduced mass with A_1 and A_2 are the mass numbers of the emitted daughter and cluster nuclei respectively. This integral can be evaluated numerically or analytically to get the half-life time of decay as

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{\ln 2}{\nu P},\tag{10}$$

where $v = \frac{\omega}{2\pi} = \frac{2E_V}{h}$ is the number of assaults on the barrier per second assuming a harmonic motion and E_V is the zero-point vibration energy and is given by the empirical formula of Poenaru et al. [52] as

$$E_{V} = Q \left\{ 0.056 + 0.039 \exp\left[\frac{(4 - A_{2})}{2.5}\right] \right\}, \text{ for } A_{2} \ge 4.$$
(11)

The zero-point vibration energy E_v vary only slightly with the mass A_2 of the cluster. This helps to predict the decay half-life with high accuracy [52].

3. Results and discussion

The barrier penetrability, decay constant, and the half-life of decay of 1-n and 2-n halo nuclei from Z = 127 - 132 were calculated using the CPPM. The CPPM is a well-established model and has been extensively used by Santhosh et al., during the past decade [53 - 58]. The model can accurately predict the alpha decay chains, heavy particle decays from elements in heavy and superheavy regions; and the predictions are in good agreement with other models such as the universal (UNIV) curve of Poenaru et al. [59], universal decay law, the analytical formula of Royer [60], Viola - Seaborg - Sobiczewski semiempirical relation [61], Gamow-like model of Zdeb et al. [62] and semiempirical relation of Hatsukawa et al. [63]. We would like to point out that we have already used the CPPM for studying the emission of 1-p and 2-halo nuclei [46, 47] and to study the emission of various exotic nuclei from superheavy elements [64, 65].

The possibility for the existence of a neutron halo structure in a nucleus can be identified by calculating the 1-n and 2-n separation energies. For a 1-n halo, 1-n separation energy is the lowest and less than 1 MeV, and for a 2-n halo nucleus, its 2-n separation energy will be the lowest and less than 1 MeV. To confirm the halo structure, we have calculated the driving potential [48] for halo nuclei with Z = 11 - 20, and the halo nuclei with Z = 3 - 10are directly taken from the work of Santhosh et al. [66], where detailed calculation of driving potentials is given. The possible 1-n halo candidates in the selected range are ¹¹Be, ¹⁴B, ^{15,17,19}C, ²²N, ²³O, ^{24,26}F, ^{29,31}Ne, ^{34,37}Na, ^{35,37}Mg, and ⁵⁵Ca. The 2-n halo candidates are ^{6,8}He, ¹¹Li, ^{12,14}Be, ^{17,19}B, ²²C, ^{27,29}F, ³⁴Ne, ³⁶Na, and ⁴⁶P. In the present work, we have studied the possibility of the emission of these halo nuclei from elements with Z = 127 - 132.

The emission of a halo nucleus from a parent is possible only if the Q-value for the decay process is positive. The Q-value is calculated using the expression

$$Q = M(A,Z) - M(A_1,Z_1) -$$
$$-M(A_2,Z_2) + k(Z_{A,Z}^{\varepsilon} - Z_{A_1,Z_1}^{\varepsilon}).$$
(12)

In this expression, M(A, Z), $M(A_1, Z_1)$ and $M(A_2, Z_2)$ represent the mass excess of the parent, daughter, and emitted halo nucleus respectively, and the term $k(Z_{A,Z}^{\varepsilon} - Z_{A_1,Z_1}^{\varepsilon})$ accounts for the screening effect of the atomic electrons [67]. For nuclei with $Z \ge 60$, k = 8.7 eV and $\varepsilon = 2.517$. For nuclei Z < 60, they are 13.6 eV and 2.408 respectively [68]. The experimental mass excess values of halo nuclei are taken from the tables of Koura - Tachibana - Uno - Yamada [69]. The mass excess values of the superheavy elements are taken from the atomic data tables of Möller et al. [70] where the mass excess value calculations are based on the finite-range droplet macroscopic model.

For light halo nuclei such as ^{6,8}He, ¹¹Li, ^{12,14}Be, and ¹⁴B, the mass excess values are high, and the Q-value of their decay is very small or even negative. From the isotopes of the parents Z = 127 - 132, the barrier penetrability and half-life for the emission of 1-n halo nuclei ¹¹Be, ^{15,17,19}C, ²²N, ²³O, ^{24,26}F, ^{29,31}Ne, ^{34,37}Na, ^{35,37}Mg, and ⁵⁵Ca; and 2-n halo nuclei ²²C, ^{27,29}F, ³⁴Ne, ³⁶Na, and ⁴⁶P were determined using CPPM. For many of the halo nuclei considered here, the half-life of decay is found to be far above the experimental limit. Only for ¹¹Be, ^{15,17}C, ²³O, ^{24,26}F, ²⁹Ne, and ³⁵Mg the half-life of decay is found to be less than or slightly above the experimental limit for the emission from the Z = 127 - 132 parents. We have calculated the decay half-life for zero momentum transfers since the angular momentum transferred in the process is extremely small ($\approx 5\hbar$) and can be neglected. The results of our calculation and more detailed discussions are given in the following paragraphs.

Since the halo is a highly deformed nuclear state, the expression for radius $R_i = R_0 A_i^{1/3}$ cannot represent the structure of the halo nuclei completely. To some extent the surface effects are included in the Süssmann central radii (C_i) ; however, R_i is a function of A_i alone. Therefore, we have considered the concept of rms radius of the halo nucleus. For a deformed halo structure, the rms radius is found to be larger than its normal radius. For our calculation purpose, the rms radius for halo nuclei $Z \le 10$ is directly taken from reference [7] and for Z = 11 - 20, we have calculated using the expression [71]:

$$\langle R_i \rangle = \langle R_i \rangle_{sph} \left(1 + \frac{5}{4\pi} \langle \beta_2^2 \rangle \right)^{1/2},$$
 (13)

where $\langle R_i \rangle_{sph}$ is the spherical radius of the halo nucleus given by $R_0 A_i^{1/3}$ and β_2 is the quadrupole deformation. The β_2 values are taken from reference [70]. We have considered about 2653 possible decays of the 1-n and 2-n halo nuclei from the isotopes of Z = 127 - 132 parents. It is to be mentioned that most of the parent nuclei are stable against the decay of 1-n and 2-n halo nuclei; however, we could find some decays that are possible with a decay half-life well within the experimental limit. Such results are given in Table. For other 1-n halo nuclei, ¹⁴B, ^{17,19}C, ²²N, ²⁶F, ^{29,31}Ne, ^{34,37}Na, ^{35,37}Mg, and ⁵⁵Ca and 2-n halo nuclei ^{6,8}He, ¹¹Li, ^{12,14}Be, ^{17,19}B, ²²C, ^{27,29}F, ³⁴Ne, ³⁶Na, and ⁴⁶P, the half-life of decays are very much above the experimental limit.

The predicted half-lives for the emission of different neutron halo nuclei from various isotopes of Z = 127 - 132 nuclei by considering the emitted nuclei as clusters and as halo

Parent	Emitted	O-value,	Barrier penetrability, P		Decay constant, λ		$\log_{10}T_{1/2}$, s	
nuclei	halo nuclei	MeV	Normal cluster	Halo	Normal cluster	Halo	Normal cluster	Halo
³³² 129	¹¹ Be	20.96	$2.011 \cdot 10^{-49}$	3.810.10-45	1.303.10-28	$2.469 \cdot 10^{-24}$	27.725	23.448
³³³ 130	¹¹ Be	21.64	$4.555 \cdot 10^{-48}$	8.939·10 ⁻⁴⁴	3.040.10-27	5.965·10 ⁻²³	26.357	22.065
³³⁴ 130	¹¹ Be	20.67	3.908.10-5	$7.474 \cdot 10^{-47}$	$2.501 \cdot 10^{-30}$	$4.783 \cdot 10^{-26}$	29.442	25.161
³³⁵ 130	¹¹ Be	22.39	9.835·10 ⁻⁴⁶	1.916·10 ⁻⁴¹	6.772·10 ⁻²⁵	$1.319 \cdot 10^{-20}$	24.010	19.720
³³⁶ 130	¹¹ Be	21.33	$6.070 \cdot 10^{-49}$	$1.159 \cdot 10^{-44}$	$3.997 \cdot 10^{-28}$	7.639.10 ⁻²⁴	27.238	22.957
³³⁵ 131	¹¹ Be	21.27	5.498.10-50	$1.089 \cdot 10^{-45}$	3.612.10-29	7.157.10-25	28.283	23.986
³³⁶ 131	¹¹ Be	22.62	8.128.10-46	$1.636 \cdot 10^{-41}$	5.649.10-25	$1.137 \cdot 10^{-20}$	24.008	19.784
³³⁷ 131	¹¹ Be	21.95	$8.524 \cdot 10^{-48}$	$1.685 \cdot 10^{-43}$	5.763·10 ⁻²⁵	$1.139 \cdot 10^{-22}$	26.080	21.783
³³⁷ 132	¹¹ Be	23.47	$4.045 \cdot 10^{-44}$	$8.377 \cdot 10^{-40}$	$2.908 \cdot 10^{-23}$	6.023·10 ⁻¹⁹	22.377	18.061
³³⁸ 132	¹¹ Be	22.46	5.070·10 ⁻⁴⁷	$1.035 \cdot 10^{-42}$	3.501.10-26	$7.148 \cdot 10^{-22}$	25.296	20.986
³²⁹ 128	²³ O	61.96	6.953·10 ⁻⁵²	$4.137 \cdot 10^{-47}$	1.167.10-29	7.168.10-26	28.733	24.985
³³⁷ 132	²³ O	64.43	7.301.10-52	3.906.10-47	1.314.10-30	7.030.10-26	29.722	24.993

We have calculated the half-life of decay of the halo nuclei by considering them as a normal spherical cluster and as a deformed nucleus with a rms radius. Figs. 1 and 2 represent the results of our calculations; the logarithmic value of half-life is plotted against the neutron number of daughter nuclei (N_d). A peak in the plot represents the neutron shell closure of the parent nucleus and a dip represents the neutron shell closure of the daughter nucleus. In the Figures we have included ¹¹Be and ²³O only; for other halo nuclei, the decay half-lives are larger than the experimental upper limit.

Fig. 1 is the plot of $\log_{10} T_{1/2}$ versus the neutron number of daughter nuclei (N_d) and shows the comparison of decay half-lives of ¹¹Be from the isotopes of Z = 127 - 132 parent nuclei when it is considered as a normal spherical cluster and as a deformed halo nucleus. It can be seen that the half-life of decay is lower for the halo structure than for the normal cluster. From Z = 129, 130, 131, and 132 parents ¹¹Be has shown finite probability for the emission with a half-life of less than 10^{30} s. Also, the plots in Fig. 1 show dips at daughter neutron numbers 190, 196, and 198, which represent the shell closure of daughter nuclei at these neutron numbers. Also, the prominent peaks at $N_d = 193$ in Fig. 1, *a* and *b* show the stability of the parent nuclei against the decay. At these peaks, the parents have a neutron number of 200 and the stability of the parent at these neutron numbers is due to the neutron shell closure of the parent. In Fig. 1, c, the peak at $N_d = 183$ represents the stability of the parent which has a neutron number $N_p = 190$; and the peak at $N_d = 193$ corresponds to $N_p = 200$. This shows that the neutron shell closure of the parent nuclei occurs at 190 and 200. In Fig. 1, d, there is a peak at $N_d = 183$ which corresponds to $N_p = 190$, and a second peak at $N_d = 201$, which corresponds to $N_p = 208$. Therefore, neutron shell closure of the parents occurs at these neutron numbers. Therefore, at neutron numbers 190, 196, 198, 200, and 208 the nucleus possesses a stable configuration due to neutron shell closure.





Fig. 1. Comparison of the half-life of emission of ¹¹Be halo nucleus from Z = 127 - 132 superheavy elements as a spherical cluster and as a deformed nucleus with rms radius. (See color Figure on the journal website.)

The comparison of decay half-lives of the ²³O nucleus as a cluster and as a halo from the selected parents is given in Fig. 2. In this case, also it is observed that the decay half-life decreases when the rms radius is considered. In plots 2, *a*, *b* and *c*, there are peaks at N_d = 185, all corresponding to stable parents with neutron number N_p = 200. This again confirms the neutron shell closure at a neutron number 200, providing additional stability to the nucleus. Further, in Fig. 2, *a*, there is a dip N_d = 188 and in Fig. 2, *f*, there is a dip at N_d = 190, which indicates

that the daughter nuclei formed in this case are stable at these neutron numbers.

For the emission of ²⁴F from the Z = 127 - 132 isotopes, the decay half-life is decreased when the rms radius of the halo structure is considered. However, we could find dominant peaks at $N_d = 185$ only for Z = 128 and 129 corresponding to a parent neutron number $N_p = 200$. Further, the emission of ²⁴F from the selected parents is less probable than ¹¹Be and ²³O.





Fig. 2. Comparison of the half-life of emission of ²³O halo nucleus from Z = 127 - 132 superheavy elements as a spherical cluster and as a deformed nucleus with rms radius. (See color Figure on the journal website.)

The prominent peaks in the plots correspond to a parent neutron number $N_p = 200$ indicating the exemptional stability of the nucleus and we predict that 200 is a neutron magic number. It is to be mentioned that calculations based on Weizsäcker - Skyrme model and finite-range droplet model have already predicted neutron shell closure at 196, 198, 200, and 202 [72]. For the other halo nuclei considered here, even the minimum decay half-life is near and far above the experimental limit. So, such plots are not included here. However, it is to be mentioned

that in the case of other halo nuclei, the decay halflife is decreased when the rms radius of the halo structure is considered. Further, in Figs. 1 and 2, one can observe an odd-even staggering in the plots of neutron number versus the logarithmic value of decay half-lives. This is due to the odd nucleon blocking effect where the odd nucleon hinders the preformation of the cluster and thereby causes an increase in the decay half-life compared to that of neighbouring isotopes. To confirm the feasibility of our study, we have made a calculation of the spontaneous fission halflife and alpha decay half-life of the parent nuclei Z = 127 - 132. The spontaneous fission half-lives were calculated using the formula of C. Xu et al. [73] given by

$$T_{1/2} = \exp\left\{2\pi \begin{bmatrix} c_0 + c_1 A + c_2 Z^2 + c_3 Z^4 + c_4 (N - Z)^2 \\ -\left(0.13323 \frac{Z^2}{A^{1/3}} - 11.64\right) \end{bmatrix}\right\},$$
(14)

where the half-life is in years, A – is the mass number, and Z – is the atomic number of the parent nucleus. The values of the constants c_1, c_2, c_3, c_4 are taken from the Ref. [73]. The alpha decay halflives were calculated using the Viola - Seaborg semi-empirical relationship given by [61]

$$\log_{10} T_{1/2} = (aZ + b)Q^{-1/2} + cZ + d + h_{\log}, \quad (15)$$

where the half-life is in seconds, the Q-value is in MeV and Z is the atomic number of a parent nucleus. The values of the constants *a*, *b*, *c*, *d*, and h_{\log} are taken from Ref. [74]. The alpha decay half-lives were also calculated using the Universal Decay Law of Qi et al. [75] given by

$$\log_{10} T_{1/2} = a Z_c Z_d \sqrt{\frac{A}{Q_c}} + b \sqrt{A Z_c Z_d \left(A_c^{1/3} + A_d^{1/3}\right)} + c,$$
(16)

where $A = \frac{A_c A_d}{A_c + A_d}$ the constants, a = 0.4065,

b = -0.4311, and c = -20.7889 are the coefficient sets determined by fitting to experiments of alpha decays [75].

The results of our calculations are shown in Fig. 3; the half-lives of spontaneous fission and alpha decay are plotted against the mass number of the parent nuclei. The half-lives of halo emission against the mass number of parent nuclei are also plotted in the same Figure. From the plot, it is clear that the spontaneous fission half-lives of the majority of the parent nuclei are much larger than their alpha decay half-life and hence they decay through alpha emission. The Z = 127, the isotopes with A > 333 decay via spontaneous fission and Z = 128, isotopes with A > 337 decay via spontaneous fission. The isotopes of parent nuclei with Z = 129, 130, 131, and 132 decay via alpha emission. Similar predic-

tions can be found in references [42] and [55]. Theoretical calculations show that the heaviest elements decay by alpha emission [76] and are experimentally verified up to ²⁹⁴118 [77]. At present, the lower limit of decay half-life for the experimental detection is ~ 10^{-6} s [77 - 79]. From Fig. 3 it is clear that the decay half-lives for halo nuclei ¹¹Be, ²³O, and ²⁴F from Z = 127 - 132 are less than the SF half-lives and are also lower than the experimental upper limit of 10^{30} s; so, these clusters can be detected in experiments. The gas-filled separators (at FLNR, RIKEN, and GSI), velocity filters (at GSI and GANIL), and the multi-reflection time-of-flight mass spectrograph, MRTOF-MS (at ISOLDE, RIKEN, GANIL, and TRIUMF) are widely used in the detection of short-lived superheavy nuclei [79, 80]. The existing experimental setups are sensitive for nuclei with half-lives roughly between tens of us up to a few hours [81]. Recently a " α -TOF" detector with a time resolution of 250.6 ± 6.8 ps was developed for correlated measurement of atomic masses and decay properties of superheavy elements [82, 83].

Now, to confirm the validity of the CPPM for applying to study the decay of halo from superheavy elements, we have plotted the Geiger - Nuttall plots (G-N) for all the 1-n and 2-n candidates considered here. Even though the G-N law is originally used to explain the alpha decay process [84] assuming Coulomb interaction, it can be applied for cluster radioactivity also [85 - 89]. Thus, we expect that the linearity of the G-N plot will not be affected by the introduction of the proximity potential along with Coulomb interaction. According to the G-N law, the logarithmic value of decay half-life follows a linear relationship with $Q^{-1/2}$ as per the equation

$$\log_{10} T_{1/2} = MQ^{-1/2} + C, \qquad (17)$$

where M is the slope of the curve and C is the y-intercept.

Fig. 4 shows the G-N plot of 1-n and 2-n halo nuclei for the decay from the isotopes of Z = 127parent nuclei. It is clear that all the plots are linear without any discontinuity. This indicates that the linearity of the plot is not altered much by the introduction of the proximity potential and confirms the validity of the G-N plot for the decay of halo nuclei. For the emission from other parents with Z = 128 - 132, we obtained similar linear plots; however, they are not included here.





Fig. 3. Comparison of the half-life of spontaneous fission, alpha emission, and halo emission from Z = 127 - 132 superheavy elements. (See color Figure on the journal website.)

Finally, Fig. 5 represents the universal curve between the logarithmic half-lives $(\log_{10} T_{1/2})$ and the negative logarithm of penetrability $(-\ln P)$ for the emission of ¹¹Be and ²³O and from Z = 127 and 128 parent nuclei. The plots are linear with the same slope. From the other parents also, we got similar linear plots. Thus, from the linearity of the G-N plot

and universal curve, it is clear that the CPPM can be used successfully for predicting the decay characteristics of various halo nuclei from superheavy elements. However, the predictions need experimental confirmation, and we hope in the near future our predictions will be verified with the availability of new experimental techniques.



Fig. 4. G-N plot of $\log_{10}T_{1/2}$ versus $Q^{-1/2}$ for various neutron halos from superheavy nuclei with Z = 127. (See color Figure on the journal website.)



-InP

Fig. 5. The universal curve for calculated logarithmic half-lives $(\log_{10}T_{1/2})$ versus the negative logarithm of penetrability $(-\ln P)$ for ¹¹Be and ²³O neutron halos from the superheavy nuclei with Z = 127 and 128. (See color Figure on the journal website.)

We would like to point out that we have considered the halo nuclei as a normal cluster as well as a deformed nucleus with a rms radius. The lowdensity distribution is not considered in the calculations. This may result in a small amount of error, but it is within the admissible limit [66]. We hope that our study will provide insight into the synthesis of the superheavy elements through the fusion reactions.

4. Conclusion

The barrier penetrability, decay constant, and decay half-life of various 1-n and 2 -n halo nuclei from Z = 127 - 132 parents were calculated using the CPPM. For ¹¹Be and ²³O the decay half-lives are found to be within the experimental upper limit. It is observed that the decay half-life is decreased when the normal radius is replaced by the rms radius of the halo structure. The neutron shell closure at neutron numbers 190, 196, 198, 204, and 208 is evident

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from the plot $\log_{10} T_{1/2}$ versus the neutron number of daughter nuclei. The prominent peak in the plots at $N_p = 200$ indicates the exemptional stability of the parent nucleus and we predict 200 as a neutron magic number. For many of the parent nuclei, alpha decay is found to be the dominant decay mode. The linearity of G-N and universal plots corresponding to the emission of halo nuclei shows the validity of G-N law in the case of decay of halo nuclei from superheavy elements.

STUDY ON THE DECAY OF Z = 127 - 132 SUPERHEAVY NUCLEI

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ДОСЛІДЖЕННЯ РОЗПАДУ НАДВАЖКИХ ЯДЕР З Z = 127 – 132 З ВИПРОМІНЮВАННЯМ 1-N I 2-N ГАЛО-ЯДЕР

Проникність бар'єрів, константа розпаду та період напіврозпаду для 1-п гало-ядер ¹¹Be, ^{15,17,19}C, ²²N, ²³O, ^{24,26}F, ^{29,31}Ne, ^{34,37}Na, ^{35,37}Mg i ⁵⁵Ca; i 2-п гало ядер ²²C, ^{27,29}F, ³⁴Ne, ³⁶Na i ⁴⁶P i материнських ядер з Z = 127 – 132 були розраховані в рамках моделі з Кулонівським потенціалом та потенціалом близькості при отриманні значень Q в краплинній моделі. Було проведено порівняння між періодами напіврозпаду при розгляді гало-кандидатів як нормальних кластерів і як деформованих утворень із відповідним середньоквадратичним радіусом. Закриття нейтронної оболонки на значеннях 190, 196, 198, 200, 204 і 208 було визначено з графіка періодів напіврозпаду залежно від числа нейтронів дочірніх ядер (*N_P*). Розрахунки періоду напіврозпаду для альфа-розпаду і спонтанного розпаду показали, що більшість материнських ядер розпадається більш імовірно через альфа-випромінювання. Графіки Гейгера - Неттолла $log_{10}T_{1/2}$ залежно від *Q*^{-1/2} та універсальні графіки $log_{10}T_{1/2}$ залежно від –lnP для випромінювання всіх 1-n і 2-n гало-ядер з материнських ядер, що розглядається тут, є лінійними, що показує справедливість закону Гейгера - Неттола для емісії гало-ядер з надважких елементів.

Ключові слова: кластерна радіоактивність, гало-ядра, надважкі елементи.

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