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COMPARATIVE STUDY BETWEEN IVBM AND IBM-2 MODELS TO CALCULATE THE ENERGY LEVELS FOR $^{162-168}_{70}$Yb ISOTOPES

This study uses the interaction vector boson model (IVBM) to identify negative parity band (NPB) energy levels in the $^{162-168}_{70}$Yb isotopes series. Simultaneously, the interacting boson model-2 (IBM-2) and the IVBM model were used to determine the ground state band (GSB) energy levels of the same isotopes. The ratios $R_{I/2}$ and $R_{(I+2)/I}$ are calculated and E-GOS (E-gamma over spin) curves are plotted to determine the properties of these nuclei in the GSB. The isotopes $^{162}_{70}$Yb, $^{164}_{70}$Yb, and $^{166-168}_{70}$Yb have different symmetries. Studies have shown that the IVBM model is more consistent with experimental results than the IBM-2 model, especially at high energy levels. This study provides a valuable comparison of results from different models, improving our understanding of the energy levels and properties of these isotopes.

Keywords: E-GOS test, interacting boson model-2, interaction vector boson model, ratio test, ytterbium isotopes.

1. Introduction

Nuclear models are important for understanding the properties of atomic nuclei since they provide a theoretical framework for interpreting experimental data and making predictions for the properties of nuclei that have not yet been studied experimentally. In this context, the shell model, collective model, interacting boson model (IBM), and the IBM with valence space (interaction vector boson model, IVBM) are some of the most important models used to describe nuclear structure [1].

Since the groundbreaking work by Mayer and Jensen [2 - 5], one of the remarkable features of the atomic nucleus that has contributed to our understanding of nuclear structure is the formation of a shell structure, which assumes that the nucleon in the nucleus is moving with an average potential created by all other nucleons in the independent-particle (or shell) model. In this model, the nucleus is divided into two parts: the core, which consists of the nucleons that occupy the lower energy levels, and the valence shell, which consists of the nucleons occupying the higher energy levels [6, 7].

The IBM is a phenomenological model used to describe the collective properties of atomic nuclei. It is based on the idea that the nucleons in the nucleus can be classified as either bosons or fermions, depending on their spin and isospin quantum numbers. The IBM proposes that bosons with angular momenta of l = 0 or 2 represent the nucleon pairs. The d-shell (l = 2), which is composed pictorially by d-bosons in an analogous manner to the shell model technique, and the simple s-shell (l = 0) are the only two shells remaining from the multitude of shells found in the shell model. The number of active nucleon (or hole) pairs outside a closed shell determines the number of bosons in an IBM system, which is based on a closed shell. As a peculiarity of the IBM, it has been successful in describing the properties of many nuclei, including those with collective rotational and vibrational modes. In this study, we have used version two of this model (IBM-2), which distinguishes between proton and neutron bosons. The IBM-2 model has been used to accurately predict the energy levels and quadrupole transition probabilities of various isotopes [8, 9].

The IVBM is a phenomenological model, meaning that it is based on empirical observations rather than fundamental principles. It is constructed using a spectrum-generating algebra called $U(6)$, which is a symmetry algebra that describes the degeneracy of energy levels in the nucleus. By using this algebraic structure and a set of bosonic creation and annihilation operators, the IVBM can describe the collective rotational spectra of nuclei with low and medium masses [10, 11]. One of the strengths of the IVBM is its ability to describe the behavior of different types of bosons, such as monopole, quadrupole, and octupole bosons. This allows the model to accurately describe the collective motion of nucleons in nuclei with different shapes and deformations, including nuclei with triaxial or octupole deformation. In recent years, the IVBM has been extended to include high angular momentum states, which has led to a better understanding of the complex nature of nuclear degrees of freedom. The model has also been applied to the study of exotic nuclei, such as those with large neutron or proton excess, where the collective behavior of neutrons plays a crucial role [12 - 16].

Overall, the IVBM is a powerful theoretical tool that has contributed significantly to our understanding of the collective behavior of neutrons in atomic nuclei. Its successes have led to the development of other boson models, such as the Generalized Seniority Model, which extends the IVBM to include more complex configurations of neutrons. Each model has strengths and weaknesses, and the choice of model depends on the specific properties of the nucleus being studied and the research question being addressed.

In this paper, we have studied some properties of ${}^{162-168}_{70}\mathrm{Yb}$ isotopes by applying the ratios $R_{I/2}$, $R_{(I+2)/I}$, and E-GOS (E-gamma over spin) methods. The calculations of the energy states for ground state band (GSB) of ${}^{162-168}_{70}\mathrm{Yb}$ isotopes were done using IBM-2 and IVBM, while the calculations of negative parity band (NPB) states were conducted just using IVBM [17]. The results were compared with the measured values for these isotopes.

2. Methodology

2.1. IBM-2

The IBM-2 Hamiltonian with the most general form has been submitted as the following [18 - 22]:

$$H = H_\pi + H_\nu + V_{\pi\nu}, \tag{1}$$

$$H = \varepsilon\left(n_{d\pi} + n_{d\nu}\right) + \kappa\left(Q_\pi . Q_\nu\right) + V_{\pi\pi} + V_{\nu\nu} + M_{\pi\nu}, \tag{2}$$

where $\varepsilon = \varepsilon_{d\pi} + \varepsilon_{d\nu}$ is the d-boson energies, $\kappa\left(Q_\pi . Q_\nu\right)$ is the proton-neutron quadrupole interaction, $M_{\pi\nu}$ is the Majorana operator, $V_{\nu\nu}$, $V_{\pi\pi}$ are the interaction of identical bosons.

2.2. IVBM

In order to determine the energy levels of the GSB and NPB of even-even nuclei, H. Ganev et al. introduced the IVBM. The interaction between the vector bosons of protons and neutrons is taken into account by the IVBM model separately [23]. This model's Hamiltonian can be expressed as follows:

$$H = aN + bN^2 + \alpha_3 T^2 + \beta_3 L^2 + \alpha_1 T_0^2. \tag{3}$$

In this scenario, the model's parameters, $a, b,$ and β_3 describe the ground state band, while α_1 and α_3 describe the octupole band. The Hermitian operator, N represents the total number of bosons, while T^2 and T_0, characterize the pseudospin, the quantum number that was introduced to distinguish between two types of vector bosons, which are the basic building blocks of the algebraic structure of the model.

The permitted values for the two bands – GSB and NPB – energy states in the IVBM model are provided by

$$E\left(I\right)_{GSB} = \beta I\left(I+1\right) + \gamma I \tag{4}$$

and

$$E\left(I\right)_{NPB} = \beta I\left(I+1\right) + \left(\gamma + \eta\right) I + \zeta. \tag{5}$$

The β parameter denotes the intensity of the rotational properties' influence, and the γ parameter denotes the intensity of the vibrational properties' influence on the nuclei. To calculate the values of the energy levels in the NPB beam, the parameters η and ζ represent an essential addition [10, 24].

2.3. E-GOS test

Plotting the ratio ($R = E_\gamma/I$) as a function of spin (I) called E-GOS, provided by [25], allows us to observe changes in the nucleus's characteristics along its excited states' identity. For each of the three limits, the relationships between R and the angular momentum I are as follows:

$$U(5):\ R = \frac{\hbar\omega}{I} \xrightarrow{I\to\infty} 0, \tag{6}$$

$$SU(3):\ R = \left(\frac{\hbar^2}{2J}\right)\left(4 - \frac{2}{I}\right) \xrightarrow{I\to\infty} 4\left(\frac{\hbar^2}{2J}\right), \tag{7}$$

$$O(6): R = \frac{E\left(2_1^+\right)}{4}\left(1 + \frac{2}{I}\right) \xrightarrow{I\to\infty} \frac{E\left(2_1^+\right)}{4}. \tag{8}$$

2.4. Backbending test

The relationship between the gamma energy E_γ and the moment of inertia ($2J/\hbar^2$) can be used to determine whether an isotope has the ability to bend backward and, if it does, where the backbending is located. The relation was given as the following [7, 8]:

$$2J/\hbar^2 = \frac{4I - 2}{E_\gamma}. \tag{9}$$

Conversely, [25] provides the relationship between the $\hbar\omega$ and E_γ:

$$\hbar\omega = \frac{E_\gamma}{\sqrt{I(I+1)} - \sqrt{\left(I-2\right)\left(I-1\right)}}. \tag{10}$$

3. Outcomes and discussion

3.1. Interaction parameters

The IBM-2 Hamiltonian parameters used in the current study to determine the energies of the positive parity low-lying levels of $^{162-168}_{70}$Yb are listed in Table 1. For $^{162-168}_{70}$Yb, N_ν changes from 5 to 7 while, N_π = 6. By fitting the experimental energy levels and allowing one parameter to vary while keeping the others constant, the IBM-2 Hamiltonian parameter values were calculated. Up until an overall fit was attained, this procedure was repeated.

Table 1. **IBM-2 Hamiltonian parameters in MeV, unless χ_π and χ_ν without units**

^A_ZX	ε_d	K	χ_π	χ_ν	C^0_π	C^2_π	C^4_π	C^0_ν	C^2_ν	C^4_ν	$\xi_1=\xi_3$	ξ_2
$^{162}_{70}$Yb	0.651	–0.165	–1.24	0.72	0.04	0.0	0.02	0.14	0.0	0.023	0.031	–0.06
$^{164}_{70}$Yb	0.49	–0.148	–1.24	0.69	0.04	0.0	0.02	–0.046	0.02	0.01	0.024	0.008
$^{166}_{70}$Yb	0.385	–0.123	–1.24	0.6	0.04	0.0	0.02	–0.7	0.1	0.03	0.01	–0.03
$^{168}_{70}$Yb	0.335	–0.105	–1.24	0.6	0.04	0.0	0.02	–0.6	0.55	0.059	0.02	0.03

The Hamiltonian diagonal was created by the computer program NPBOS [26]. One nucleus's energy spectrum can theoretically be fitted by independently varying each parameter. Calculations show that almost six quantities, ε, κ, χ_π, χ_ν, C^L_π, and C^L_ν, determine the structure of the spectra. In general, these quantities could be affected by both the neutron boson number N_ν and the proton boson number N_π. Using microscopic calculations from [27] as a guide, we presume that only ε and κ are dependent on both N_π and N_ν, while χ_π is dependent only on N_π which are constant for all isotopes of any particular element, and χ_ν are dependent on N_ν. As a result, χ_π is the same for a group of isotopes. As well as, the coefficients C_L are taken as C^L_π, as a function of N_π and C^L_ν, as a function of N_ν respectively, meaning that the proton-proton interaction will only depend on N_π and the neutron-neutron interaction will depend on N_ν [11]. Therefore, one can correlate a lot of experimental data thanks to parameterization. There is a similarity between our parameters and those in Ref. [28], as ε, which has the highest effect, starts with the highest value and gradually decreases, as well as the proximity of the values to each other. Likewise for the rest of the parameters.

3.2. Energy levels

IBM-2 and IVBM models mentioned before having been used to calculate the positive parity ground state energy levels of the isotopic chain $^{162-168}_{70}$Yb in major shell 82, and also, we have calculated the negative parity ground state energy levels of the same chain by using the IVBM model. The results are depicted in Fig. 1. The Figure shows a detailed comparison with experimental data. We can observe that the IVBM model can give a better fit with the experimental results than the IBM-2 model especially at high spins, as one of the drawbacks of the IBM-2 model is that it cannot provide satisfactory results for energy levels with spins higher than 12.

As shown in Fig. 1 the agreement between experimental [23] and theoretical results is quite good, and the general features are well reproduced; it is clear that the IVBM model was more consistent with experimental results, particularly for high spin, in addition to the ability of this model to calculate energy levels for negative parity band as well.

3.3. Dynamic symmetry testes

Numerous tests can be used to predict the nucleus's dynamic symmetry, whether it is vibrational, rotational, or something in between [17, 29]. In this Section, we will look at the ratios tests, such as $R_{I/2}$ and $R_{(I+2)/I}$. Table 2 shows the limiting values for $R_{I/2}$, where I = 4, 6, 8, and 10 for the dynamical symmetry $U(5)$, $O(6)$, and $SU(3)$; in comparing them with the experimental and calculated ratios we could note that the $^{162}_{70}$Yb isotope belongs to the gamma unstable limit $O(6)$ and $^{164-168}_{70}$Yb chain belongs to the rotational limit $SU(3)$. While from the first energy level $E\left(2^+_1\right)$ test we could say that $^{162-164}_{70}$Yb are translations between $O(6)$ - $SU(3)$, and the other $^{166-168}_{70}$Yb are $SU(3)$.

Table 3 shows the other type of ratio test $R_{(I+2)/I}$, where we can note that the $O(6)$ limit is the closest limit for $^{162}_{70}$Yb isotope. For $^{164}_{70}$Yb the ratios $R_{4/2}$ and $R_{6/4}$ show that the isotope belongs to $SU(3)$, while the ratios $R_{8/6}$ and $R_{10/8}$ show that the isotope is $O(6)$. The rest isotopes $^{166-168}_{70}$Yb are $SU(3)$. This result is similar to what S. H. Ibrahem (2023) concluded in his research paper [30], where he said that, all the $^{160-166}_{70}$Yb isotopes are between $O(6)$ and $SU(3)$ limit except $^{166}_{70}$Yb which is near the $SU(3)$ limit, due to the increase in the number of its bosons than the rest of the isotopes.

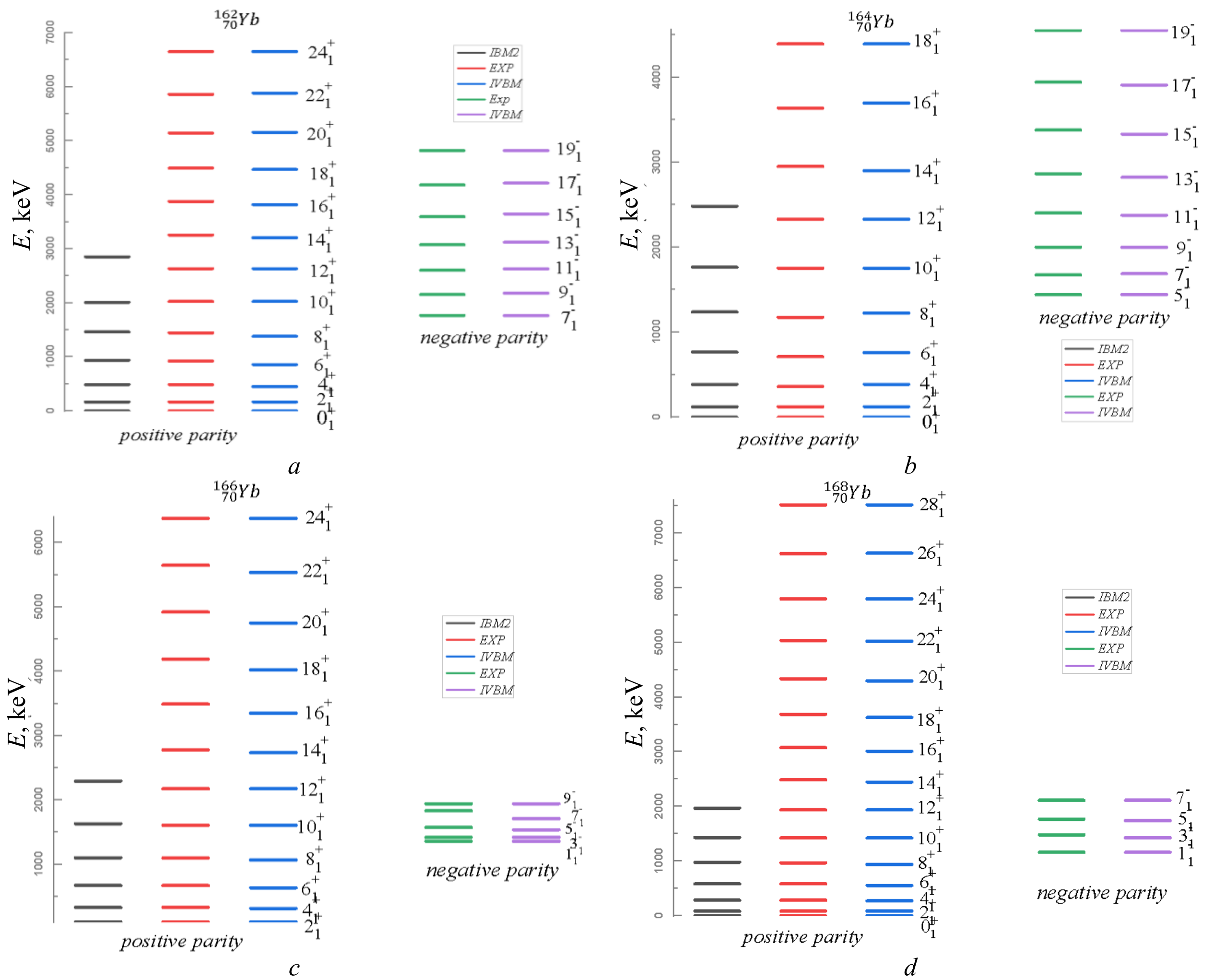

Fig. 1. Comparison between experimental and calculated energy levels for $^{162\text{-}168}_{70}$Yb isotopes. (See color Figure on the journal website.)

Table 2. **Comparison between typical and calculated values for energy ratios $R_{I/2}$**

Symmetry		$E(2_1^+)$	$R_{4/2}$	$R_{6/2}$	$R_{8/2}$	$R_{10/2}$
$U(5)$		500	$2 \le R_{4/2} \le 2.4$	3	4	5
$O(6)$		300	$2.4 \le R_{4/2} \le 2.7$	4.5	7	10
$SU(3)$		100	$3 \le R_{4/2} \le 3.3$	7	12	18.33
$^{162}_{70}$Yb	IBM-2	166	2.946	5.723	9.23	13.4
	Exp	166.7	2.923	5.543	8.67	12.1
	IVBM	166.7	2.714	5.143	8.28	12.1
$^{164}_{70}$Yb	IBM-2	120	3.242	6.642	11.2	16.9
	Exp	123.3	3.127	6.165	9.92	14.2
	IVBM	123.3	2.922	5.766	9.53	14.2
$^{166}_{70}$Yb	IBM-2	101	3.238	6.881	11.73	17.80
	Exp	102.4	3.228	6.523	10.72	15.68
	IVBM	102.4	3.067	6.204	10.41	15.68
$^{168}_{70}$Yb	IBM-2	88	3.193	6.523	10.97	16.25
	Exp	87	3.294	6.728	11.15	16.38
	IVBM	87	3.138	6.414	10.83	16.38

Table 3. **Comparison between typical and calculated values for energy ratios $R_{I+2/I}$**

Symmetry		$R_{4/2}$	$R_{6/4}$	$R_{8/6}$	$R_{10/8}$
$U(5)$		2	1.5	1.33	1.25
$O(6)$		2.5	1.8	1.56	1.43
$SU(3)$		3.33	2.1	1.71	1.53
$^{162}_{70}$Yb	IBM-2	2.946	1.943	1.614	1.455
	Exp	2.923	1.896	1.564	1.400
	IVBM	2.714	1.895	1.611	1.465
$^{164}_{70}$Yb	IBM-2	3.242	2.049	1.686	1.513
	Exp	3.127	1.971	1.609	1.433
	IVBM	2.922	1.973	1.653	1.492
$^{166}_{70}$Yb	IBM-2	3.238	2.125	1.705	1.517
	Exp	3.228	2.021	1.644	1.462
	IVBM	3.067	2.023	1.678	1.507
$^{168}_{70}$Yb	IBM-2	3.193	2.043	1.681	1.482
	Exp	3.294	2.042	1.657	1.469
	IVBM	3.138	2.044	1.688	1.513

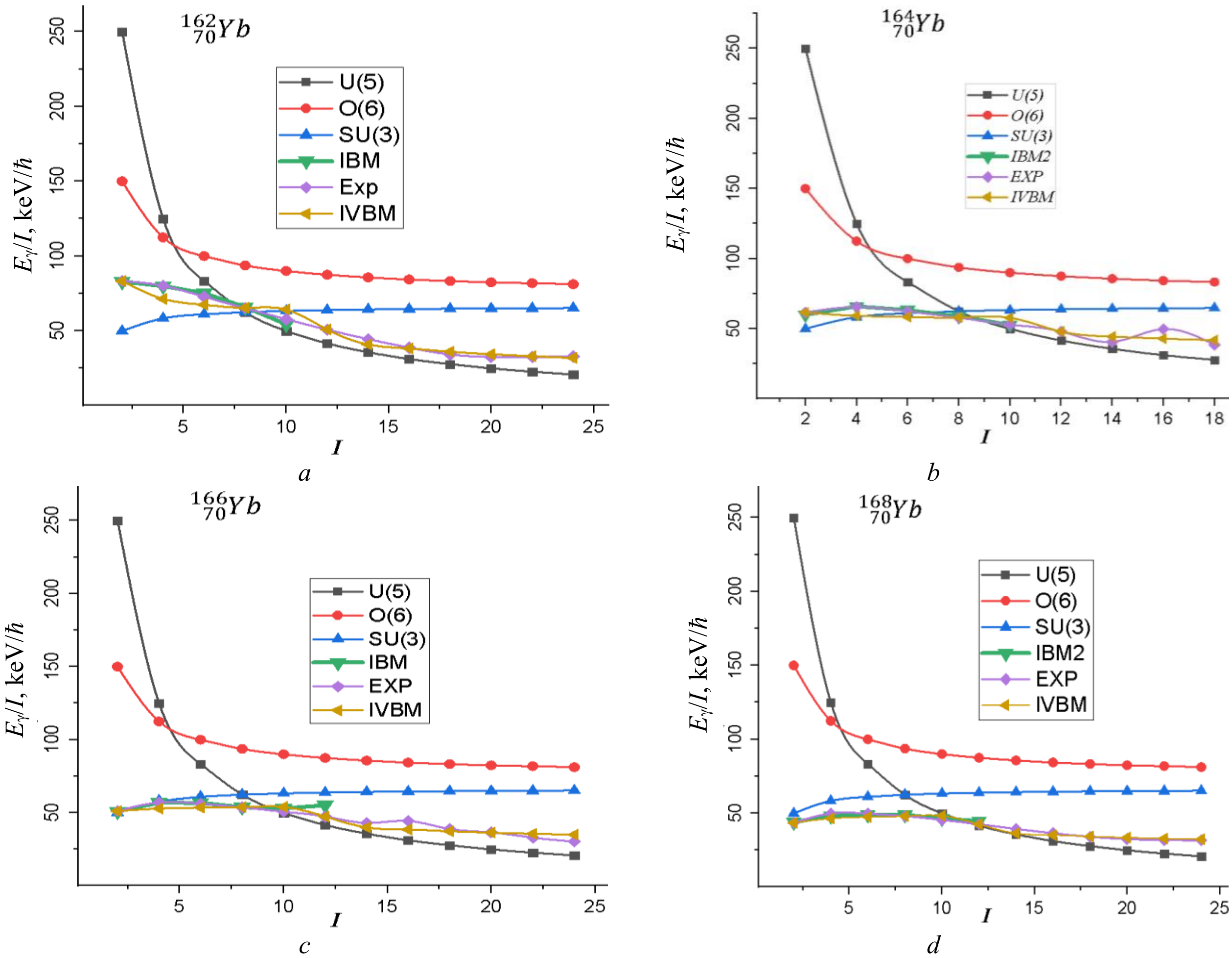

Fig. 2. E-GOS plot for the yrast sequence in $^{162-168}_{70}$Yb.
(See color Figure on the journal website.)

Since Tables 1 and 2 didn't provide all of the information regarding the nucleus's properties at its various excited states which are subject to change, Fig. 2 depicts the E-GOS of the measured gamma energy. We could notice from Fig. 2, *a* that the $^{162}_{70}$Yb isotope shifts from a gradual decrease to a fairly rapid decrease and back to a gradual decrease, which could confirm that the $^{162}_{70}$Yb isotope has γ-unstable symmetry $O(6)$. While Figs. 2, *b* and *c* show that the curves of the isotopes $^{164}_{70}$Yb and $^{166}_{70}$Yb begin with a slight rise at the first exited spin 4^+ and then turn to a gradual decline, which indicates the presence of a transitional phase shift (*SU*(3) - *O*(6)) for those isotopes. As for Fig. 2, *d*) we find that the curve of $^{168}_{70}$Yb isotope begins to rise more than the curves of the previous two isotopes and continues to rise until reaching the spin 10^+ and then gradually decreases after that, which indicates the rotational *SU*(3) symmetry for this isotope.

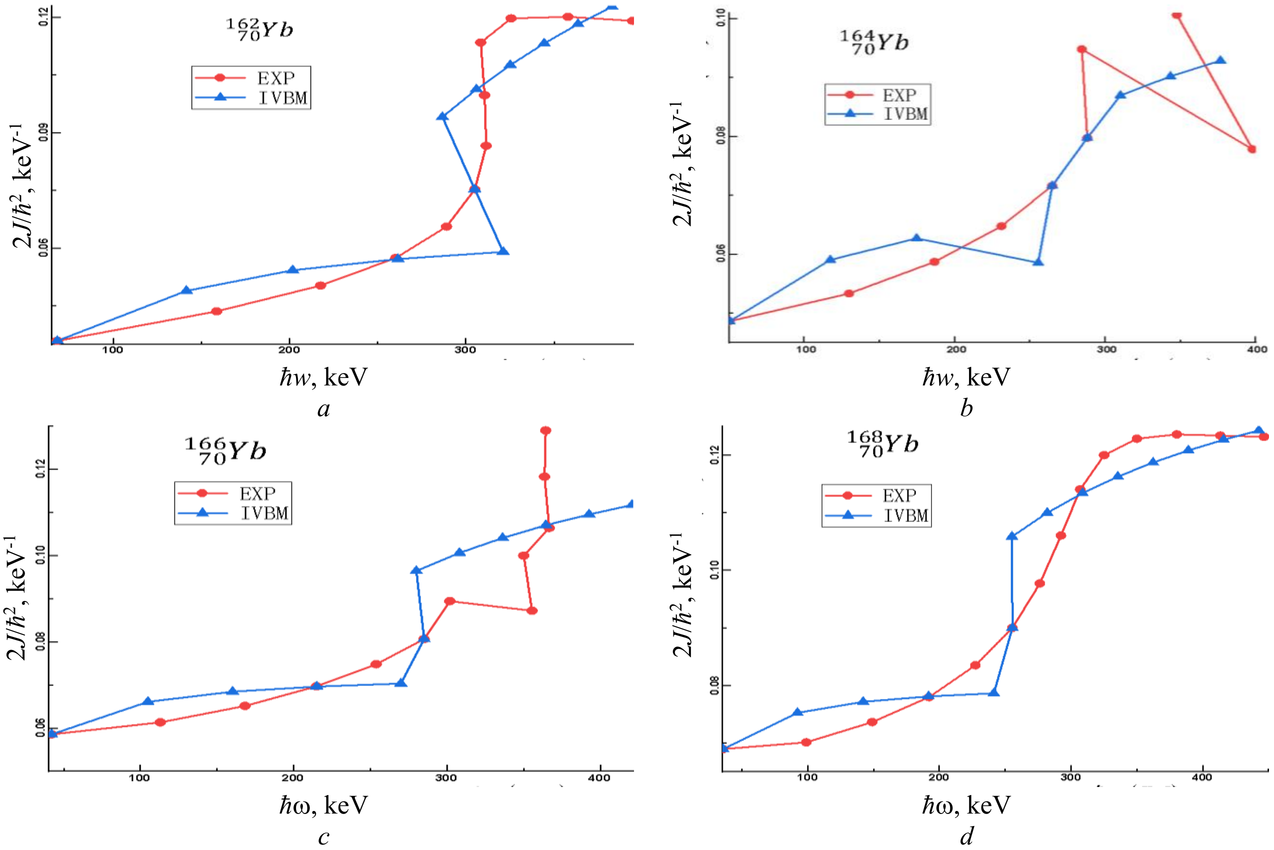

Fig. 3. Backbending plot for the yrast sequence in $^{162-168}_{70}$Yb.
(See color Figure on the journal website.)

For more information about the Yb nuclei, we tested the presence of backbending in this chain of isotopes, and as shown in Fig. 3, the $^{162}_{70}$Yb isotope has good backbending while for isotopes $^{164}_{70}$Yb and $^{166}_{70}$Yb a backbending could barely be seen, but in the case of $^{168}_{70}$Yb an unbinding is remarked. These results are somewhat similar to those in Ref. [31] for the chain $^{164-168}_{70}$Yb.

4. Conclusion

In conclusion, for $^{162-168}_{70}$Yb isotopes with neutron numbers from 92 to 98, the positive parity energy levels are computed by IBM-2 using NPBOS program and IVBM using MATLAB program, while the negative parity energy levels were computed using IVBM only. The analysis demonstrates that the outcomes of these models and the available experimental data agree fairly well. The GSB of the aforementioned isotopes has been described by the ratios $R_{I/2}$ and $R_{(I+2)/I}$. As well as, plotting and comparing the energy gamma over spin E-GOS curves of the GSB for $^{162-168}_{70}$Yb nuclei with the ideal limits of vibrational, rotational, and soft cases are done. According to this study, the $^{162}_{70}$Yb isotope has the *O*(6) property, the $^{164-166}_{70}$Yb have the transformation property in between *O*(6) - *SU*(3) and $^{168}_{70}$Yb isotope has the rotational property. The backbending test has been done also where a clear backbending in isotope $^{162}_{70}$Yb, a slight backbending in isotopes $^{164}_{70}$Yb and $^{166}_{70}$Yb, and no backbending has been found in isotope $^{168}_{70}$Yb.

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ПОРІВНЯЛЬНЕ ДОСЛІДЖЕННЯ МОДЕЛЕЙ IVBM ТА IBM-2 ДЛЯ РОЗРАХУНКУ ЕНЕРГІЙ РІВНІВ ІЗОТОПІВ $^{162–168}{}_{70}$Yb

У цьому дослідженні використовувалася модель взаємодіючих векторних бозонів (IVBM) для визначення енергій рівнів, що належать до зони негативної парності (NPB), для ізотопів $^{162–168}_{70}$Yb. Разом з тим, моделі взаємодіючих бозонів (IBM-2) та IVBM використовувалися для визначення енергій рівнів для зони основного стану (GSB) тих самих ізотопів. Для визначення властивостей цих ядер у GSB розраховувалися відношення $R_{I/2}$ і $R_{(I+2)/I}$ та криві E-GOS (енергія гамма-кванта залежно від спіну). Ізотопи $^{162}_{70}$Yb, $^{164}_{70}$Yb і $^{166–168}_{70}$Yb мають різну симетрію. Дослідження показали, що модель IVBM більш узгоджується з експериментальними даними, ніж модель IBM-2, особливо для високоенергетичних рівнів. Це дослідження дає корисне порівняння результатів, отриманих з різними моделями, покращуючи наше розуміння енергетичних рівнів і властивостей цих ізотопів.

Ключові слова: E-GOS тест, модель взаємодіючих бозонів 2, модель взаємодіючих векторних бозонів, тест відношення, ізотопи ітербію.

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